

Optimal Card Collecting Strategy

for Magic: The Gathering

Zackary Bedwell & Tony Favata

October 21st 2026

Introduction

Imagine for us

- You open a pack of collector cards, hoping to find that last card you've been looking for, only to get another duplicate. As your collection grows, it becomes harder to find a rare card you do not already own.
- Your wallet hurts, and there's only so much spending you can do to find that last rare card.

This raises a mathematical question intriguing to Zack and I:

Q: Is there a purchasing strategy that minimizes the expected total cost of collecting all the different rare cards?

The Complex to Concrete

The Stronghold Expansion - Magic: The Gathering

- In this expansion, there are 143 total card types. Of these, 44 are **rare**, another 44 are *uncommon*, and the remaining 55 are *common*. We will only focus on the **rare** cards.
- Each expansion pack consisting of 15 Magic cards is guaranteed to have only 1 distinct rare card. That means for each pack opened, 1 out of the 15 cards will be one of our 44 distinct rares.

Haircut

- Our author, *Dr. Robert A. Bosch*, used a PDF to determine a matrix of probabilities when pulling rares from a pack. This would create a 45x45 Matrix, exceeding Zach's and my patience.
- Instead of studying the full example with 44 distinct rares, we're going to shorten this to 4 rares, then compare it to the general solution.

Our Example: Collecting Four Rare Cards

Instead of displaying a 45×45 matrix, let's trim the fat.

- Suppose we want to collect all four possible rare card types:

$$R_1, \quad R_2, \quad R_3, \quad R_4.$$

Properties of Packs

- 1 Each pack **MUST** contain exactly one rare card.
- 2 Each rare card type is equally likely to be pulled:

$$P_r(R_i) = \frac{1}{4}, \quad i = 1, 2, 3, 4.$$

- 3 Packs are independent, so duplicates are possible.

States and Notation

Since packs are independent, the first pack could contain R_1 , and the second pack could also contain R_1 .

What is the state?

The state is the number of **different rare card types** we own at that point in time. In this example, we have 5 discrete states:

0, 1, 2, 3, 4.

Example

If we had R_1, R_1 , and R_3 , we would be in **state 2**, because we have two *different* rare card types. The second R_1 is a duplicate and does not increase our state.

Probability Matrix

Rows: current state.

Columns: state after one pack.

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

NOTES:

- ① Rows and columns correspond to states 0, 1, 2, 3, 4.
- ② Each row adds to 1: it lists all possible outcomes.
- ③ One pack can add at most one different rare card type.
- ④ State 4 is **absorbing**: the collection stays complete.

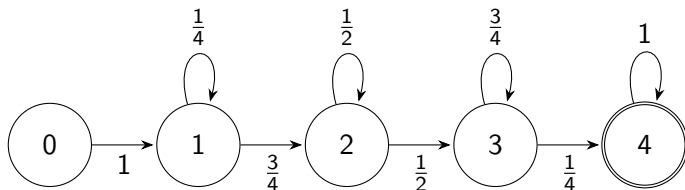
Why is this matrix useful?

P describes the transition probabilities after one pack. P^2 describes them after two packs. P^n describes them after n packs.

State Diagram

Navigation of our “State Diagram”

Each arrow represents opening one pack. Its label gives the probability of that transition. A new rare moves us forward, while a duplicate keeps us in the same state.



Example: State 2

We own two of the four rare card types. There is a $\frac{1}{2} = 50\%$ chance of a duplicate and a $\frac{1}{2} = 50\%$ chance of pulling a new rare.

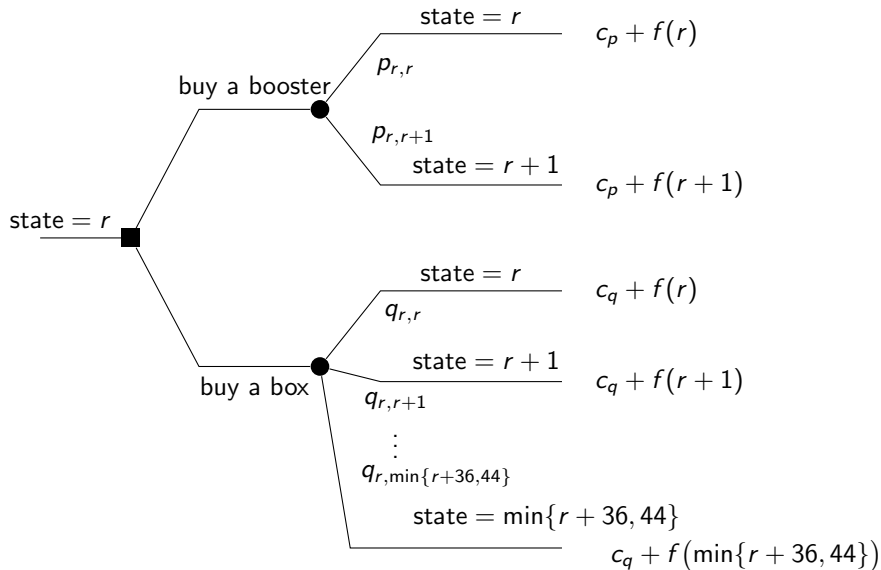
The Cost Function

- 1 Now that we've figured out how to find all of our 4 rares, lets now find the optimal financial strategy.
- 2 This is the connecting factor from statistics to financial strategy. This will show how to go from state $r = 0$ to $r = 4$.

Context Parameters

- $p_{r,r+1}$ where r : is the current state and $r + 1$: is the final state of a singular booster pack (p).
- $q_{r,r+1}$ where r : is the current state and $r + 1$: is the final state inside of a booster box (q), consisting of 2 booster packs (p)
- $c_p = \$2.95$: This is the fixed price of a singular booster pack in 1998 for Magic the gathering.
- $c_q = \$5.35$: We will assume a price here since some other parameters changed. This will define the single cost of a booster box.

The Cost Function



A Concrete Example of the Cost Function

Suppose we own three different rares and need just one more.

Option 1: buy a singular pack (p)

We pay \$2.95 for our pack. There are two outcomes:

- A duplicate rare would leave us in state 3.
- A new card finishes the collection.

$$2.95 + \frac{3}{4}f(3) + \frac{1}{4}f(4) = 2.95 + \frac{3}{4}f(3).$$

Option 2: buy a Two-Pack box (p^2)

Likewise, there are two outcomes for the booster box:

- Both cards are duplicates. $\Rightarrow p(r) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$.
- Otherwise, the collection is complete.

$$5.35 + \frac{9}{16}f(3) + \frac{7}{16}f(4) = 5.35 + \frac{9}{16}f(3).$$

A Concrete Example of the Cost function

Now, since this function is an ordered pair, choose the purchase with the smaller expected total cost:

$$f(r) = \min \left(\begin{array}{l} c_p + f(r)p_{r,r} + f(r+1)p_{r,r+1}, \\ c_q + f(r)q_{r,r} + \sum_{r'=r+1}^{\min\{r+36,44\}} f(r')q_{r,r'} \end{array} \right) \quad (1)$$

Now, substituting in $r=3$ to solve our ordered pair:

$$f(3) = \min \left(2.95 + \frac{3}{4}f(3), 5.35 + \frac{9}{16}f(3) \right)$$

Reverse Engineering

Compare buying packs repeatedly with buying boxes repeatedly, until we find the missing card.

Individual Packs

Let C_p be the expected remaining cost using packs.

$$C_p = 2.95 + \frac{3}{4} C_p.$$

$$\frac{1}{4} C_p = 2.95$$

$$C_p = \$11.80$$

Two-Pack Boxes

Let C_q be the expected remaining cost using boxes.

$$C_q = 5.35 + \frac{9}{16} C_q.$$

$$\frac{7}{16} C_q = 5.35$$

$$C_q \approx \$12.23$$

$$f(3) = \min\{11.8, 12.23\} = \$11.80$$

In summary, with one rare card type missing, buy individual packs.

Working Backward: State 2

We now know $f(3) = 11.8$ and $f(4) = 0$. Use these to solve state 2.

Individual Packs Until Our State Improves

After reaching state 3, follow its optimal strategy.

$$C_p = 1 + \frac{1}{2}C_p + \frac{1}{2}(11.8) \quad \implies \quad C_p = 13.9$$

Two-Pack Boxes Until Our State Improves

The state-2 row of P^2 gives probabilities $\frac{1}{4}, \frac{5}{8}, \frac{1}{8}$ of ending in states 2, 3, 4.

$$C_q = 5.35 + \frac{1}{4}C_q + \frac{5}{8}(4) + \frac{1}{8}(0)$$

$$C_q = \frac{4}{3}(5.35 + 2.5) \approx 10.43$$

$f(2) = \min\{13.9, 10.43\} \approx \10.43

The Optimal Purchasing Strategy

Repeat the same backward calculation for states 1 and 0.

State r	Best Purchase	Expected Cost $f(r)$	Other Purchase $f(r)$
4	Stop	\$0.00	\$0.00
3	One pack	\$11.8	\$12.23
2	Two-pack box	\$10.43	\$17.70
1	Two-pack box	\$20.61	\$20.90
0	Two-pack box	\$23.23	\$23.56

The Optimal Strategy

- If the collector is missing (2) or more rares, purchasing a booster box will increase their chances.
- With the contrary, once the collector hits $r = 3$, they should purchase singular packs until they reach their final state of $r = 4$.

Time Constraints and other flavors

- Like Zach and I stated, this was a hefty amount of information to cram into 15 minutes. This was an interesting strategy to concretely study the card collecting strategy.
- Some other flavor that are mentioned briefly were store exclusive discounts, coupons, or BOGO deals.

Questions, comments, and concerns are welcome

The Goat

Bosch, R. (2017). Optimal card-collecting strategies for Magic: The Gathering. Digital Commons at Oberlin. <https://doi.org/10.2307/2687095>