

Counting Triangles from a Diced Larger Triangle

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Research Question

How many triangles of any size will be created inclusive of the original triangle when we divide its interior with h evenly spaced horizontal lines and v evenly spaced vertical lines?

Introduction

Have you ever wanted to test your IQ quickly with just one question? If so, try this problem and try to solve it in under 3 minutes! Pop-ups on advertisements on childhood games and logic puzzles all advertise this same IQ test through a singular question, but these questions have stuck with me over time. This also continuously pops up in life now, as I scroll through Instagram reels with random Reddit storytimes with background videos that consist of styrofoam being placed on top of evenly spaced vertical and horizontal heated wires (this sounds so random but I'm being so serious).

One of the questions proposed by a logic game app asked to find the number of triangles there were in the image, which was cut up by horizontal and vertical lines. In the moment, with few horizontal and vertical lines cutting the shape, it was easy to count the triangles as a young child. However, this sparked the research question above as I try to find a general formula to count the number of triangles there are with less and less specific factors. My main goal is to heal that younger curious child and finally find a way to count the triangles in a more "advanced" way that could actually prove my actual IQ rather than a random app generated one.

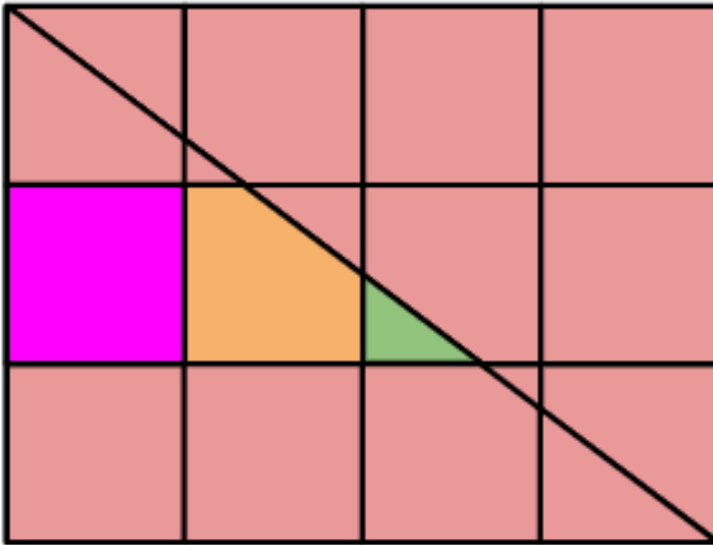
History

When copy and pasting my exact question into google, the first link that pops up is a YouTube video titled "'98% Fail' - How Many Triangles Are There? Viral Bollywood Puzzle." This question and similar variations have been circulating India as a brain teaser, but the roots of this problem don't go back as long as one might expect. Most questions circulating the internet claim that the history of this problem lasts only in the past 25ish years or just this century, but much of the actual history of this problem is likely lost and spread amongst different ancient civilizations.

Most of the mathematical background is from my mini research paper, however I had absolutely no background for that paper. A quick explanation of this is the formula

$$\# \text{ of triangles total} = 1 + h + v + \text{ceiling}[(h + 1)/(v + 1) - 1]$$

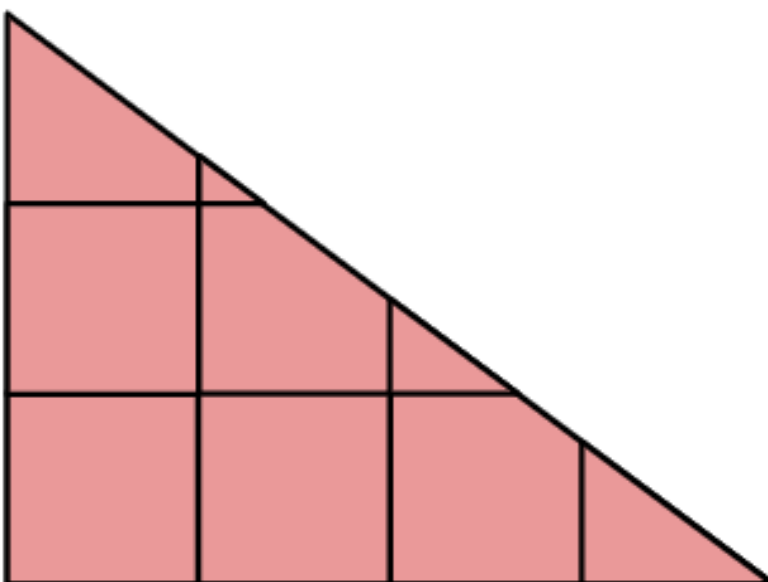
This is the formula we use to find the number of triangles when it is a right triangle with bases along the vertical and horizontal axis. For this formula you have to first choose a side to count from and remove the outsides and repeat the ceiling function for the number of layers along that side. For example, in this image



We are counting along the horizontal axis and we remove the outside layer (which would be the bottom in this case because of the horizontal axis) and you have 2 more horizontal layers, so repeat the ceiling function twice. This will be helpful when we break down different types of triangles and have to use prior formulas to find the number of triangles in different types of triangles.

The other math background to consider is similarity as this will be used a lot in proving my formulas and how different ways to solve for the number of triangles can vary. This is later explained in the investigated I also must explain the ceiling function as this is used to round any decimal up to the next integer which is necessary in this function as it relies on the ratio between different sides and will not be a whole number when we are counting triangles that don't take a full unit, such as the green. The ceiling function makes sure that we still count this green triangle. This is all the background math I had before conducting my investigation.

Mathematical Background



To go more in depth, let's start over with the normal rectangle. In this, you can count the number of spaces in the triangle to find out the total which would be 9 triangles. Also, you find that whenever you cut by only horizontal lines or only vertical lines, the total number of triangles is just $1 + h$ or v as parallel lines cut by a transversal have corresponding congruent angles and therefore similar triangles are created. However, when you combine horizontal and vertical lines this is different. We can start with

$$\begin{aligned} \text{triangles} &= 1 + h + v + ? \\ 9 &= 1 + 3 + 2 + ? \\ ? &= 3 \end{aligned}$$

So to find the missing value to calculate by, we're going to use $\text{ceiling}[x\{(h + 1)/(v + 1)\} - 1]$.

We're also going to repeat this ceiling function for the number of layers in the triangle when there are at least 2 parts after negating one of the bases. This sounds kind of crazy so let's go through this example.

We're choosing to use the first layer, so $x = 1$ and whatever value is greater when plugging in for h and v and in this case $v > h$ so we're doing

$$\begin{aligned} &\text{ceiling}[x\{(v + 1)/(h + 1)\} - 1] \\ &\text{ceiling}[(4/3) - 1] \\ &1 \end{aligned}$$

Looking at the triangle, we are calculating the number of uncounted triangles in the top layer horizontally. We subtract 1 because this accounts for the base spaces, like the spots along the axis' and we already counted that in the equation. We're dividing $(v + 1)/(h + 1)$ because this is the side length ratio of each triangle ever created in this original triangle via similarity, so this division finds the dimensions of the entire top layer. We multiply by x , the number layer we are using, as this adjusts the ratio for the specific layer as another layer multiplies this ratio by 2 to get double the vertical and horizontal length, so you just multiply to get the right dimensions of this new layer. So, $(v + 1)/(h + 1) - 1$ is one of the dimensions of the uncounted area of the top layer and since we have $v > h$, this will count the horizontal length.

Looking at the triangle again, this makes sense as the $4/3$ is the horizontal length of the top layer and subtracting 1 gets you $1/3$, which is the horizontal length of the small little triangle to the right of the base side. Lastly, a ceiling function is needed as even though the triangle is smaller than 1 unit, it still counts, so we need to round up to the integer.

Lastly, we need to repeat this for all the horizontal layers besides the base one, so we have the middle section which is layer 2.

$$\begin{aligned} &\text{ceiling}[x(4/3) - 1] \\ &\text{ceiling}[2(4/3) - 1] \\ &\text{ceiling}[(8/3) - 1] \\ &2 \end{aligned}$$

This finds that the middle layer creates 2 extra triangles which is right! Since the last horizontal layer is the base, we don't need to count it because it's already accounted for. Therefore, we use the formula

$$\text{triangles} = 1 + h + v + \text{ceiling}[x\{(h + 1)/(v + 1)\} - 1]$$

and repeat the ceiling function until all layers are accounted for to find the total number of triangles. Lastly, let's check how right we are

$$9 = 1 + 3 + 2 + 1(\text{layer 1}) + 2(\text{layer 2})$$

$$9 = 9 \checkmark$$

Investigation

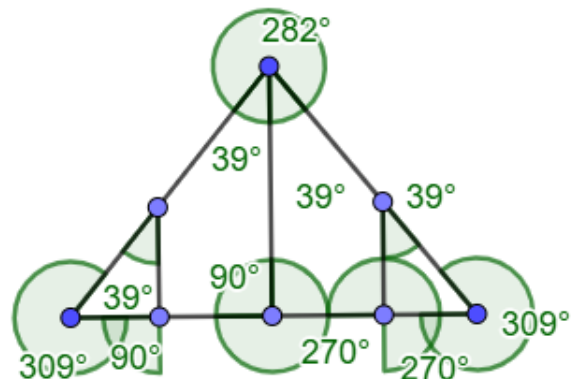
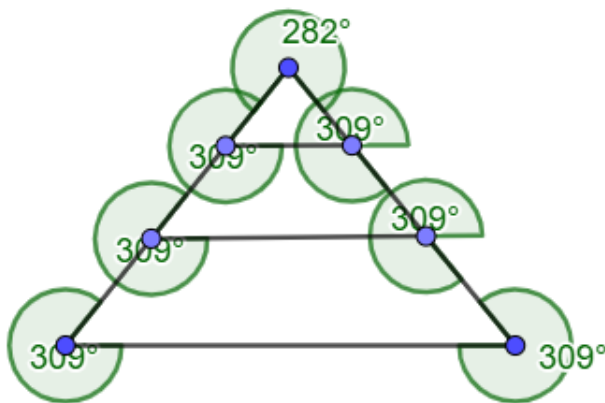
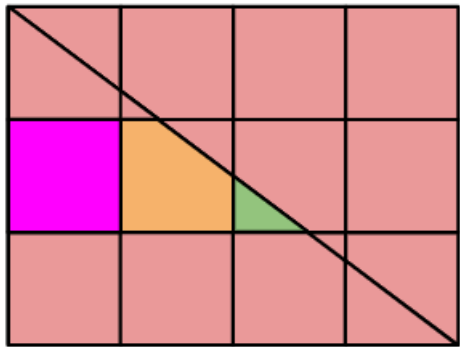
From my mini research paper, I have found that the recursive formula

$$f(1) = 1 + h + v$$

$$f(n) = f(n - 1) + \text{ceiling}[(h + 1)/(v + 1) - 1]$$

works for right triangles given that $f(n)$ is the total number of triangles counted n and we repeat the formula until all horizontal slices but the bottom most one (on a leg of the triangle) are counted.

Is it possible to use a similar formula to find a formula for different types of triangles? First, I tried this with an acute triangle.



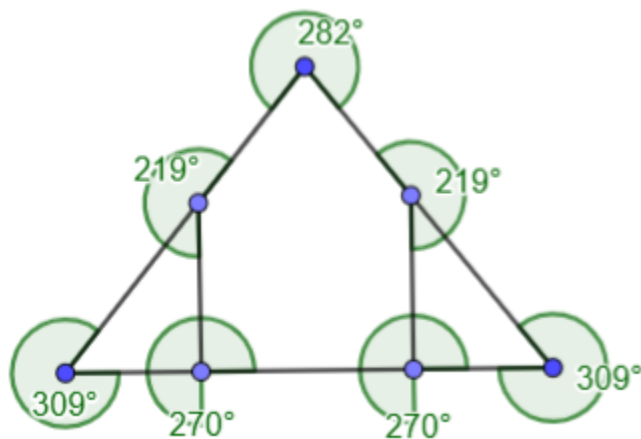
Through this, we can see that the triangles are both acute as the outer angles are greater than 270° , so the inner angles are less than 90° , making the triangles acute. On the left, I cut the triangle with horizontal lines while on the right, I cut them with vertical lines. We can see that even if the lines are not evenly spaced, the total number of triangles is

$$f(n) = 1 + h$$

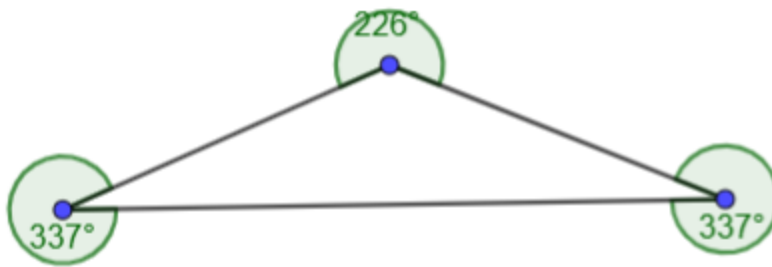
like for the right triangles.

This is because of similar triangles. Because my example has a horizontal line as the base of the acute triangle, which we can see as it has the same angle degree, any additional horizontal lines that cut through the triangle are parallel to the base triangle. When two parallel lines are cut by a transversal, which is a line not parallel to the other two lines and intersects the both of them, congruent corresponding angles are created. In this, the base and new horizontal line are the parallel lines while the transversal is the non-base side of the original triangle. Since the base angles for the triangle are equal, then the new triangle we formed from the horizontal line added is similar to the original triangle as transversal on both sides create congruent corresponding angles for the new triangle and the reflexive theorem states that the angle is the same as itself - since we use the same third angle for both triangles, this is also the same. Now that we understand that the triangles created from horizontal line segments will always be similar to each other, then we can understand that the formula works as each h new line will create a new triangle to add to $f(n)$ given that we count the original triangle too. This also means that the horizontal line segments don't need to be evenly spaced to follow this formula → maybe this could be used for an extension?

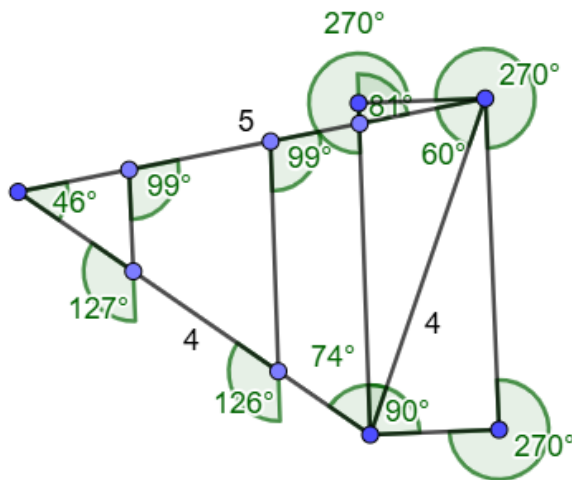
For the vertical line segments, this is not the same. The vertical line segments are not parallel to anything, but perpendicular to the base. Additionally, we can see that you can only count new triangles until the third point on either side of the original triangle, as creating a shape from new vertical lines on each side of this point gives you a five-sided polygon, not a triangle. The point at the top creates 2 different sides on each side of it, so this is slightly different from before.



As we can see here, the middle section could not be a triangle anymore because choosing 2 vertical lines on opposite sides of the third point removes the common side and creates an extra side on top as there is one side on either side of the third point, and adds another side as we are making the bounds of the shape between two vertical lines rather than include a point. Therefore, this variable may make it harder to develop a formula that works to solve for the total number of triangles.



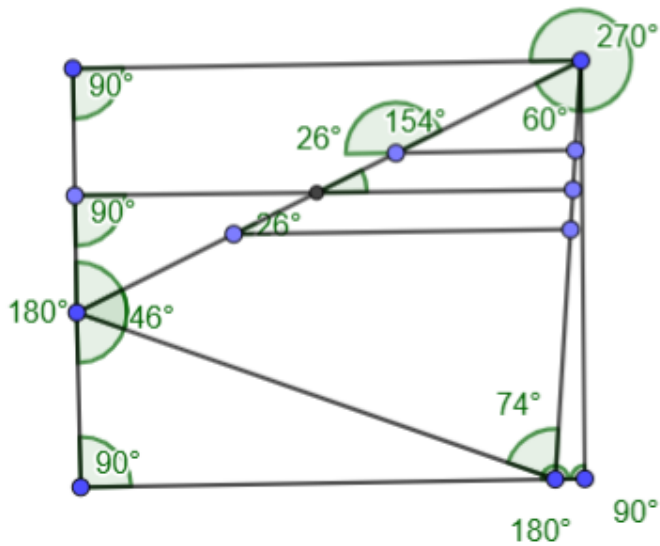
I realized that the same issue arises with an obtuse triangle as well. The main difference in how you count the new triangles is from your side lengths and the angle of your sides - like how my base always is horizontal, if you change the base angle to something else, counting it would be different.



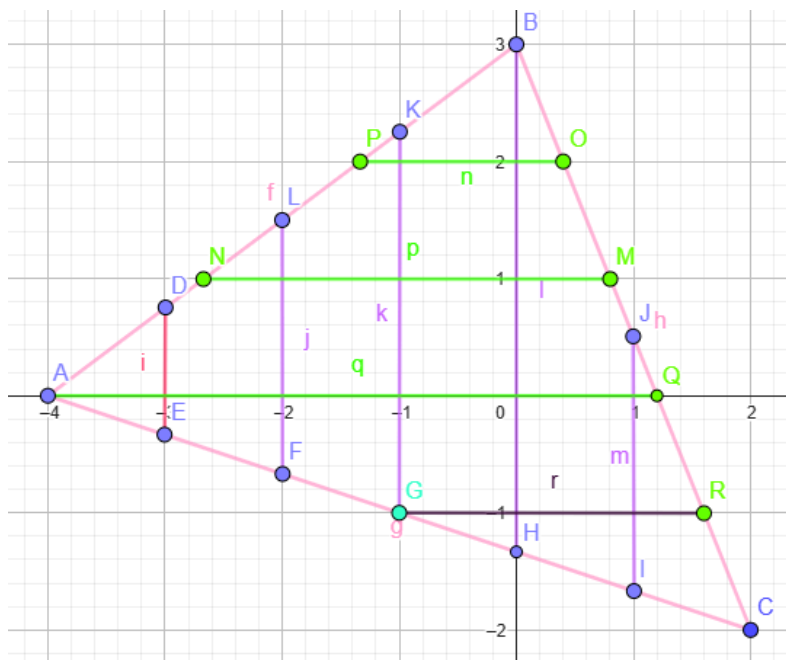
So, I realized from this, that the question really changes only when you rotate the triangle, no matter what type of triangle it is. The number of triangles that one can make when you cut parallel to a base is completely different from just having any horizontal or vertical line added.

So, from this we can see that all vertical cuts from the third point, which is 74° , to a different point almost act like an independent triangle and the line vertical from the third point acts as a parallel base for this new mini triangle. This separates the original triangle into 2 smaller triangles based on the altitude to help count the number of triangles easier.

Does this process work the same when adding horizontal lines?



It seems as if this is the same when you add horizontal lines instead of vertical lines. How might this change when you combine both horizontal and vertical lines?



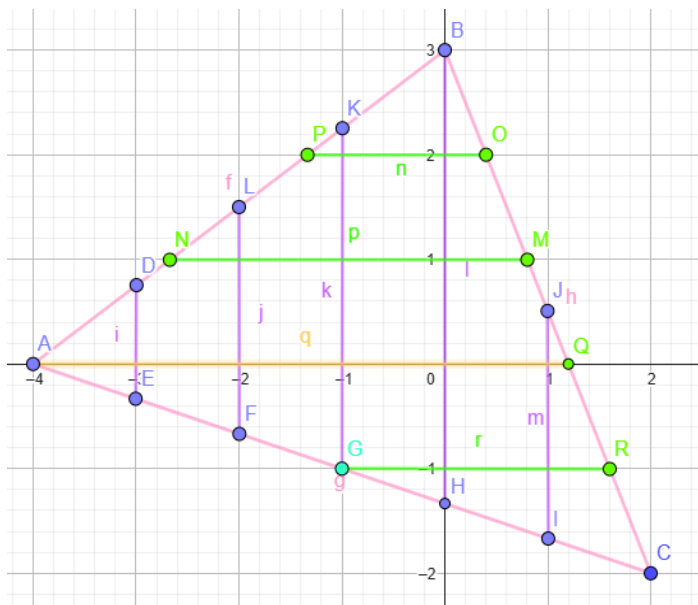
I realized through this new graphing feature on Geogebra that the number of triangles can be counted in different ways. The first method could be to split the triangle up into four equal parts by the x and y axis. If the axes are not provided, you split them up by choosing the middle point when looking horizontally and cutting the triangle vertically from that point and then choose the middle point vertically and cut the triangle horizontally from that point. Then we get 4 triangles which look like the ones we saw in the right triangle when the base layer is removed. However, this undercounts the number of triangles that are created as the triangles can be different types of triangles, not just right.

If you look at the section of triangle under the brown line, you can see that there are 3 triangles. Because of the vertical line cuts, if you focus on the cyan point and go to the right, you can see a small triangle and then a larger triangle inclusive of the small triangle and the larger triangle encompassing the other two. These triangles are created based on the number of vertical lines that cut it up, which is 2. Isolating this smaller acute triangle such that there are only vertical lines cutting through creates $v + 1$ triangles where v is the number of vertical lines.

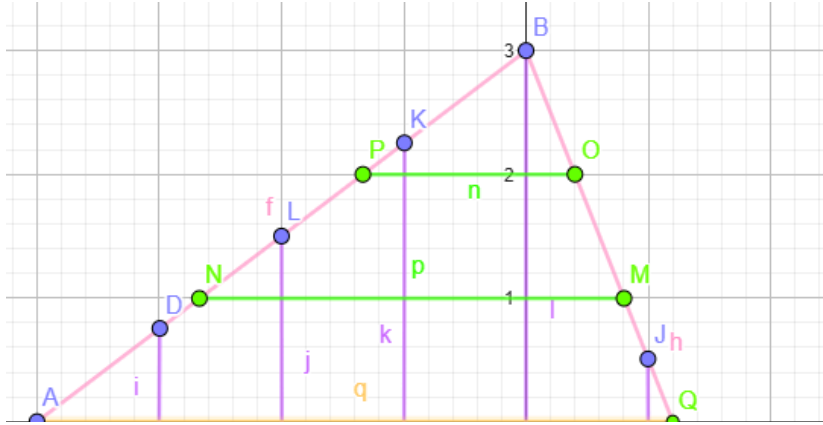
This is the same for horizontal when you look at the triangle to the left of the red line. The number of smaller triangles is $h + 1$ where h is the number of vertical lines. This ties back to the case of the right triangle as horizontal and vertical lines cut triangles simply until they are combined.

Also, before we discussed how cases would be different when the side of the triangle was vertical or horizontal, but through the cutting by axes, discussed before, you can see that this is actually used to count the number of triangles whether or not the sides are vertical or horizontal. Given that you only cut by one axis, then you have two halves with sides parallel to either horizontal or vertical and you would just be doing a summation. However this needs to be repeated for the other axis. You don't have to multiply by anything crazy either as you see in the picture that once you try to count past these axes, you get a different sided polygon, not a triangle anymore.

Let's look at some examples when you cut along one of the axes:



For this, we are going to cut along the x-axis which is colored in orange and focus on the top half. This is going to be called Part 1. I noticed a difference between the right and acute triangles in that counting the number of polygons left in the shape due to the vertical and horizontal cuts is actually NOT equal to the number of triangles actually created.

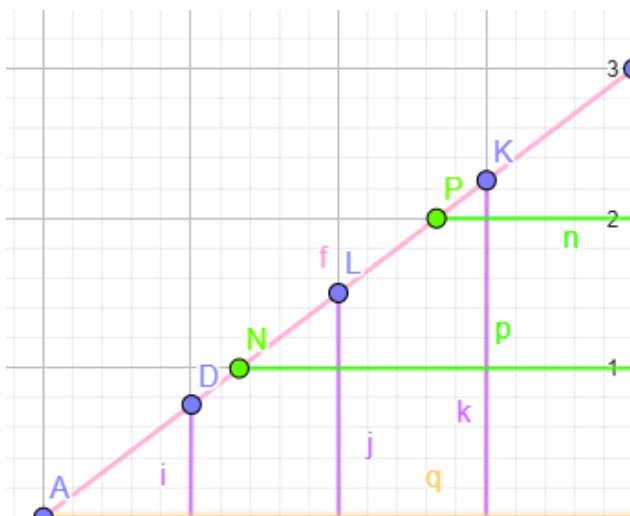


First, for the first layer there are 4 total triangles. Going from point P to the right there are 3, as there is one small and then a bigger one that inhales the small as well and then on the other side of the vertical cut there is one more triangle. Because of the triangles on both sides of the cut, you can combine these to get an extra triangle. $2 + 1 + 1 = 4$ *total triangles* from level 1. The two are for the two triangles to the left of the vertical line and the first 1 is the one to the right of this line and the 1 at the end is when you combine both sides to get one largest triangle of the level.

Second, you can count the second layer for a total of 5 triangles. There is the isolated one on the left. Taking the next vertical segment, one triangle contains 3 spaces and the next vertical segment triangle contains 5 spaces. Now back on the other side, there is just one extra triangle. Yet again, you have 1 largest triangle of the given space because the split of triangles is side-by-side, so they combine to form one triangle. Adding these all up, you get $1 + 1 + 1 + 1 + 1 = 5$ *total triangles* from level 2.

For the third you can repeat this process to get 7 total triangles for this row. In total for the top section of the triangle, you have $7 + 5 + 4 = 16$ *total triangles*.

It seems as if you can use the ratio and ceiling function from the right triangles to count these triangles too with the additional step of adding + 1 for every layer there is as to count the largest triangle for that layer.



Looking at my conclusions for the right triangle, I counted the number of triangles using

$$\# \text{ of triangles total} = 1 + h + v + \text{ceiling}[(h + 1)/(v + 1) - 1]$$

Remember that we repeat the last term for each level of the triangle that has shapes besides those on the base layer and that the h and v can be switched such that the fraction is always equal to or greater than 1. Just doing the left side first, we have $h = 2$ and $v = 3$, so when you plug this in you get

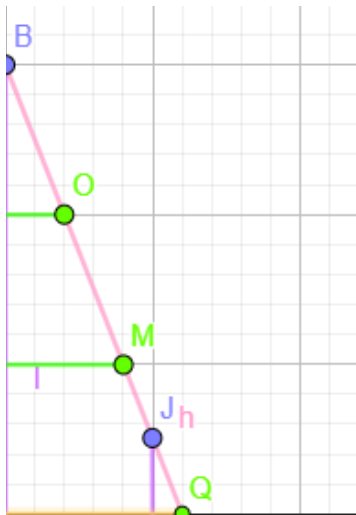
$$\# \text{ of triangles total} = 1 + 2 + 3 + \text{ceiling}[(3 + 1)/(2 + 1) - 1]$$

$$\# \text{ of triangles total} = 6 + 1 + \text{ceiling}[2\{(3 + 1)/(2 + 1)\} - 1]$$

$$\# \text{ of triangles total} = 6 + 1 + 2$$

$$\# \text{ of triangles total} = 9$$

First, I plugged in the original h and v values to get to 7 total triangles, but you still have to count the second level of the triangle, so we redo everything in the ceiling function multiplied by 2 however, because this is the second level, so since the height of the triangle multiplied by 2, the whole fraction is multiplied by 2 to compensate. Then, you just finish out the equation without doing the bottom layer, because it is already accounted for in the $+ h + v$, which also explains why we are allowed to subtract one from the fraction in the ceiling function as that one represents the outermost vertical side of the triangle. In the end you get a total of 9 triangles on the left side.



Let's calculate this for the right and see if they match up. For this side we have $h = 2$ and $v = 1$.

$$\# \text{ of triangles total} = 1 + 2 + 1 + 1$$

$$\# \text{ of triangles total} = 4$$

In this, though it looks like there are 2 levels that we should count, we only count starting with the level that has at least 1 space outside of the base sides, so for this we would not have to use the ceiling function because the triangle only consists of the base sides and no additional triangles on the inside of this if that makes sense.

Therefore, our total number of triangles for the right side is 4. When we add up the left and right sides, you get

$$\# \text{ of triangles total} = 4 + 9$$

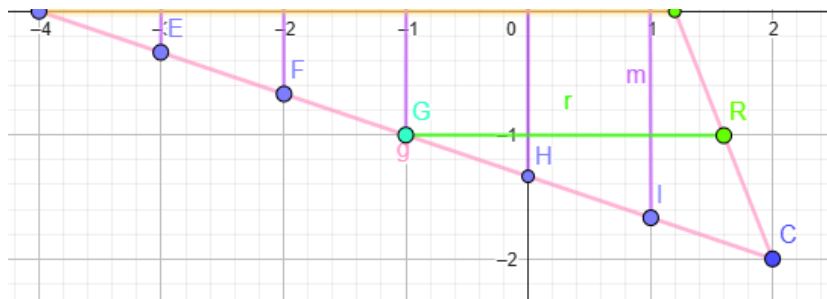
$$\# \text{ of triangles total} = 13$$

But the actual total number of triangles is 16. We can get this by adding 3 or $(h \text{ or } v) + 1$. You can determine whether to use your h or v value by seeing which line cuts the triangle in two and choosing the opposite. For this, we have a vertical line cutting the triangle (to get left and right), so we choose $h + 1 = 3$ and add this to the total number of triangles to get 16.

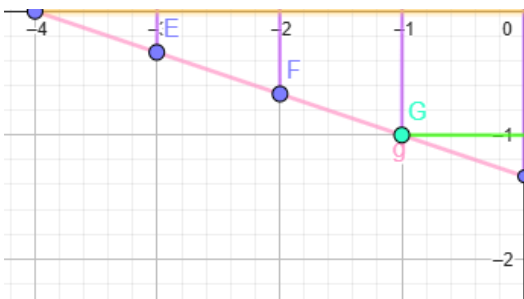
$$\# \text{ of triangles total} = \text{Left} + \text{Right} + (h \text{ or } v) + 1$$

$$\# \text{ of triangles total} = 9 + 4 + (2) + 1$$

$$\# \text{ of triangles total} = 16$$



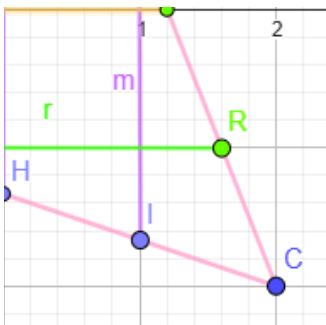
Now, does this also work for the bottom half of the triangle (the part under the yellow line)? I counted 9 total triangles without the formula, but when using the formula, you have $h = 1$ and $v = 3$ for the left side of the bottom triangle.



$$\# \text{ of triangles total} = 1 + 3 + 1 + \text{ceiling}[(3 + 1)/(1 + 1) - 1]$$

$$\# \text{ of triangles total} = 1 + 3 + 1 + 1$$

$$\# \text{ of triangles total} = 6$$



For the right side of the triangle, you have $h = 1$ and $v = 1$.

$$\# \text{ of triangles total} = 1 + 1 + 1 + cf[(1 + 1)/(1 + 1) - 1]$$

$$\# \text{ of triangles total} = 3$$

Then, you can plug it into the final equation:

$$\# \text{ of triangles total} = \text{Left} + \text{Right} + (h \text{ or } v) + 1$$

$$\# \text{ of triangles total} = 6 + 3 + (1) + 1$$

$$\# \text{ of triangles total} = 11$$

So this time it was actually overcounted and you can see that in my examples, this only works when the length of the bases are corresponding with their respective perpendicular line cutters - 1. What I mean by that is in the top half of the triangle, we see that the side lengths of the bases on the left side are 4 and 3 with 3 vertical lines and 2 horizontal lines respectively, which is - 1 from the side length. This makes sure that the lines are evenly spaced out, however since the top is evenly spaced, the bottom triangle no longer is because the altitude point lies outside the triangle.

We've already seen that the formulas to find one type of triangle can help find a different type of triangle via the right triangle formula to help find acute triangles to an extent. I realized that just counting up the different spaces in the bottom triangle works to count up the triangles, so 9 spaces, 9 triangles. I also realized that you can look at an obtuse triangle as if it's a right triangle and for each base side that has a non-integer side length, extend it so that they are integers and don't look at the triangle like 2 separate triangles, but as one right triangle on the bottom.

Then you use the original formula with $h = 1$ and $v = 5$:

$$\# \text{ of triangles total} = 1 + 1 + 5 + ceiling[(5 + 1)/(1 + 1) - 1]$$

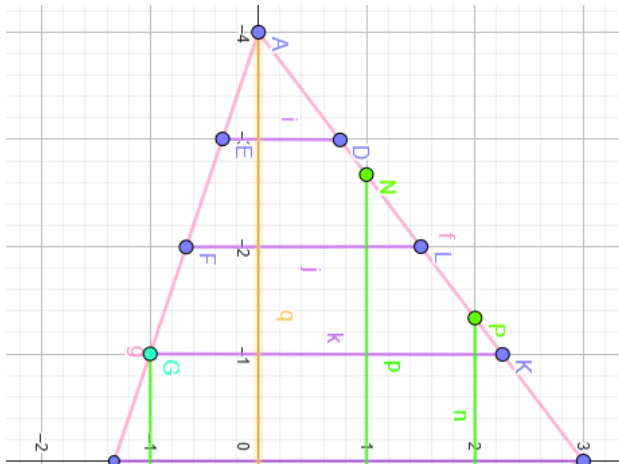
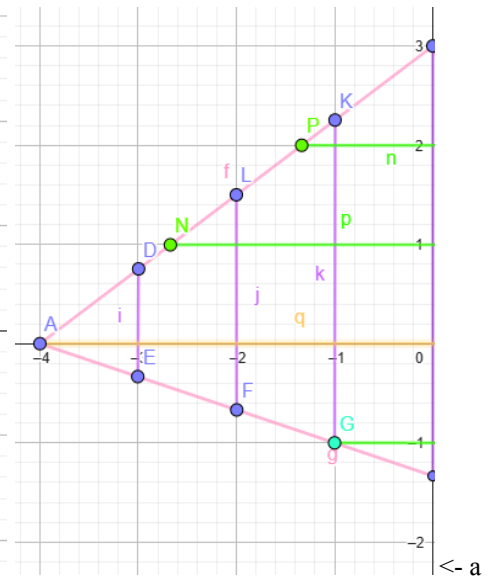
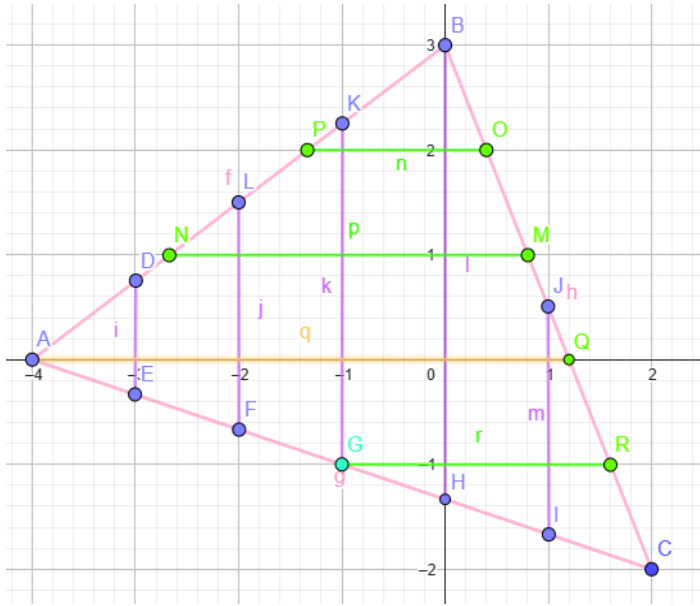
$$\# \text{ of triangles total} = 1 + 1 + 5 + 2$$

$$\# \text{ of triangles total} = 9$$

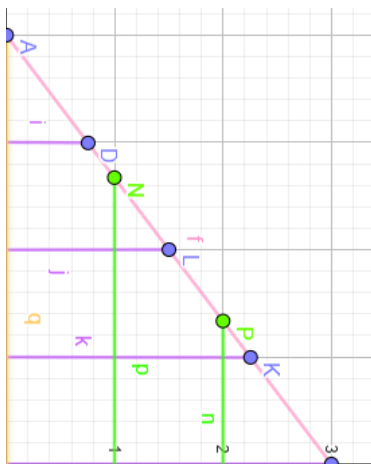
To wrap this up, when your obtuse triangle has a base side of either horizontal or vertical, such that the obtuse angle lies along this base, you can use the original right triangle formula to find the number of triangles it contains.

We've found ways to count the number of triangles for all 3 types of triangles but only when a base lies on either the vertical or horizontal axis or both. Can we observe anything else that helps us find the total number of triangles inclusive on the original triangle no matter how the triangle is formatted?

I noticed in my investigation that when I cut the triangle along the x-axis, I am undercounting the triangles that take from both the top and bottom halves of the triangle and are cut along the y-axis. Therefore, we can count the triangles when cut vertically with the same process as before. This is Part 2.



<- b - rotated the left side (new top triangle)



<- c - right side when cut by axis

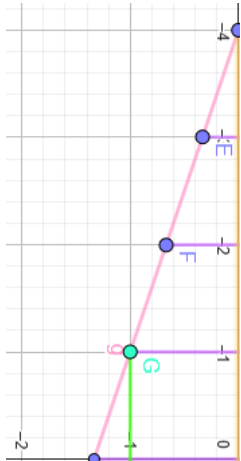
For the left side of the triangle (a), the circumcenter is inside the triangle, so it's the same process as the top half of the triangle. The way I'm looking at it, we rotate the triangle such that the left side is now on the top (b). The new top has a right side (c) with $h = 3$ and $v = 2$.

$$\# \text{ of triangles total} = 1 + 2 + 3 + \text{ceiling}[(3 + 1)/(2 + 1) - 1]$$

$$\# \text{ of triangles total} = 6 + 1 + \text{ceiling}[2\{(3 + 1)/(2 + 1)\} - 1]$$

$$\# \text{ of triangles total} = 6 + 1 + 2$$

$$\# \text{ of triangles total} = 9$$



You can notice that we've seen this number before. That's because we're counting the same triangle from the left section of the top triangle in Part 1 -> This will help us later!

Now we have to count the left side with $h = 3$ and $v = 1$.

$$\# \text{ of triangles total} = 1 + 1 + 3$$

$$\# \text{ of triangles total} = 5$$

Similar to the situation with the right side of the top triangle in Part 1, because this triangle doesn't have any spaces created besides those on the base/outside, you can't use the ceiling function, and you just add the original by the number of horizontal lines and the number of vertical lines.

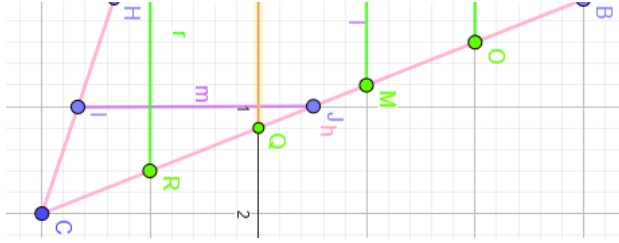
Then we see that the triangle is being cut into the left and right by a vertical line, so you choose the opposite, the horizontal, and add $h + 1$ to the sum of the left and right sides.

$$\# \text{ of triangles total} = \text{Left} + \text{Right} + h + 1$$

$$\# \text{ of triangles total} = 5 + 9 + 3 + 1$$

$$\# \text{ of triangles total} = 18$$

So the formula says that we have 18 triangles on the top and this tracks!



Let's do this bottom now using the obtuse -> right triangle formula. In this we have $h = 1$ and $v = 4$.

$$\# \text{ of triangles total} = 1 + 1 + 4 + \text{ceiling}[(4 + 1)/(1 + 1) - 1]$$

$$\# \text{ of triangles total} = 1 + 1 + 4 + 2$$

$$\# \text{ of triangles total} = 8$$

The formula says we have 8 triangles on the bottom and this also tracks!

How can we combine all the triangles from Parts 1 and 2 together to evaluate the entire triangle? Some think that we can just add all values from Parts 1 and 2 together, but that overcounts the number of triangles created as we count the same triangles multiple times depending on what axis cuts the triangle. For me, I counted a total of 31 triangles in the larger triangle. Then I realized, when you cut by a specific axis and count the triangles on either side of this axis, you wouldn't cut the triangle by the other axis as well, you just add the number of h or $v + 1$ and you choose between the h or v by choosing the opposite as the line that divides the triangle in two.

For example, if we count from Part 1, we add the top and bottom and then h or $v + 1$. For this we have $v = 5$ inclusive of the y -axis.

$$\# \text{ of triangles total} = \text{top/left} + \text{bottom/right} + h \text{ or } v + 1$$

$$\# \text{ of triangles total} = 16 + 9 + 5 + 1$$

$$\# \text{ of triangles total} = 31$$

Finally, we found the number of triangles total and this matches! Let's make sure it works when cutting along the y -axis too. We have $h = 4$ for this and the values from Part 2.

$$\# \text{ of triangles total} = \text{top/left} + \text{bottom/right} + h \text{ or } v + 1$$

$$\# \text{ of triangles total} = 18 + 8 + 4 + 1$$

$$\# \text{ of triangles total} = 31$$

Guys guess what?! This also works! Let's summarize the formulas we used.

Conclusion

Right and obtuse triangles when along an axis:

$$f(1) = 1 + h + v \quad f(n) = f(n - 1) + \text{ceiling}[(h + 1)/(v + 1) - 1]$$

Remember that we repeat this for n layers that include shapes besides those on the base sides. Acute triangles when along an axis first split along the altitude using a perpendicular line from the base axis to create 2 right triangles and then add h or $v + 1$ where you choose h or v based on the opposite axis that you cut the acute triangle with (cut with vertical choose horizontal)

$$f(n) = \text{left} + \text{right} + h \text{ or } v + 1$$

Any triangles can be split along one axis first, calculate the top/left and then the bottom/right using the previously explained formulas to get two halves of the triangle and then add $h \text{ or } v + 1$ and choose h or v by the same method as before as this finishes counting the remaining triangles.

I am not sure that there is a way to completely combine all of this into one formula, but now we understand the process to solve for the number of triangles with different conditions.

Application/Extension

This problem could be used in architecture, with blueprints of a house or accent walls that are covered in mirrors or windows. Really, any one surface that is covered in triangles or squares relates to this problem as you can figure out how many spaces there are mathematically. I would have really liked to create a general formula to count all the triangles, but I could not figure it out. Also, I attempted to relate this with Pythagorean's Theorem, but could not figure out how to, so the extension questions are:

How can this problem be solved with Pythagorean's Theorem?

What happens when we remove more parameters: equally spaced lines, only being horizontal and vertical, lines that fully cut through the shape?

How could we apply this to different polygons besides a triangle?

Would they follow a pattern when they are regular polygons vs. not?

Other methods to solve this problem?

References

- 1) MindYourDecisions. "'98% Fail' - How Many Triangles Are There? Viral Bollywood Puzzle." *YouTube*, 10 Apr. 2018, www.youtube.com/watch?v=OLJDnpWr8TY. Accessed 2 June 2025.
- 2) Wong, Violet. "Given a right triangle with a horizontal leg and a vertical leg, how many triangles of any size will be created inside the triangle when we divide its interior with h equally spaced horizontal line segments and v equally spaced vertical line segments?" 14 Jan. 2025. Student Paper