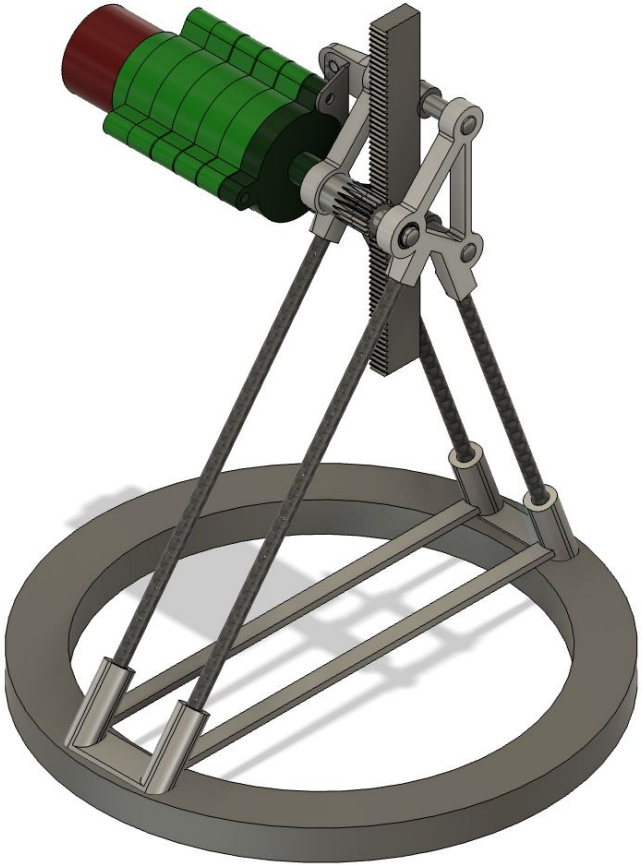
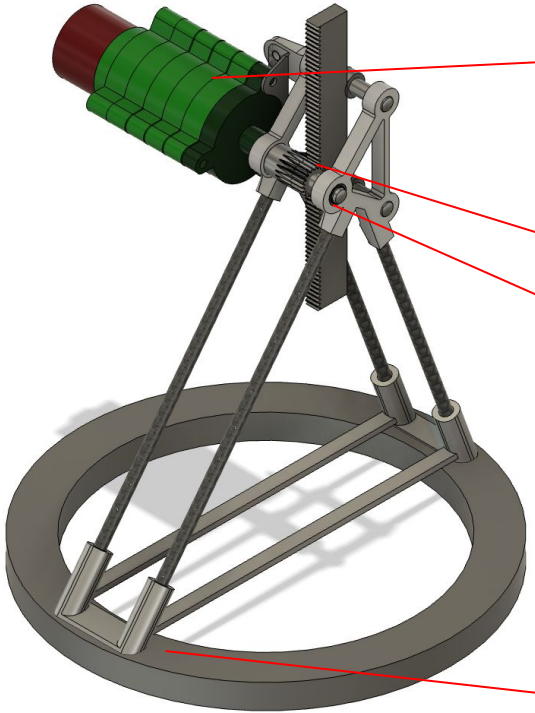


Rack-and-Pinion Impulse Machine

Mari Zampa

Design



Motor at the top of the assembly

Pinion directly attached to the rack, which is supported by rollers to ensure stability

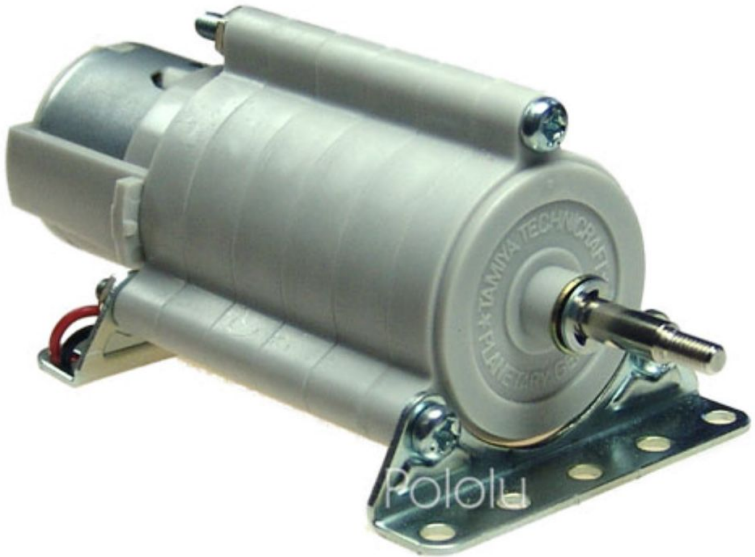
Contact Points

Contact between the pinion–rack–diver
- ○ Reduce friction loss
- ○ All contact points/motor are collinear

Legs

Angle minimized (vertical)
Double sided tape
Reduce bending

Motor Set Point Selection



Core Relationships

Impulse: $J = F \times t$

Force (from Power): $F = P/v$

Substituted Impulse: $J = (P/v) \times t$

Characteristics

Peak Operating Speed: $\omega_{peak} = \omega_0/2$

Motor Torque: $\tau = K_t \times I$

Power Output: $P = \tau \times \omega = K_t \times I \times \omega$

Summary

Net Mechanical Power: $P_{mech} = V \times I - I^2R$

Total Impulse Delivered: $J = \int F\,dt = \int (P/v)\,dt$

Set Point Selection: Max Power

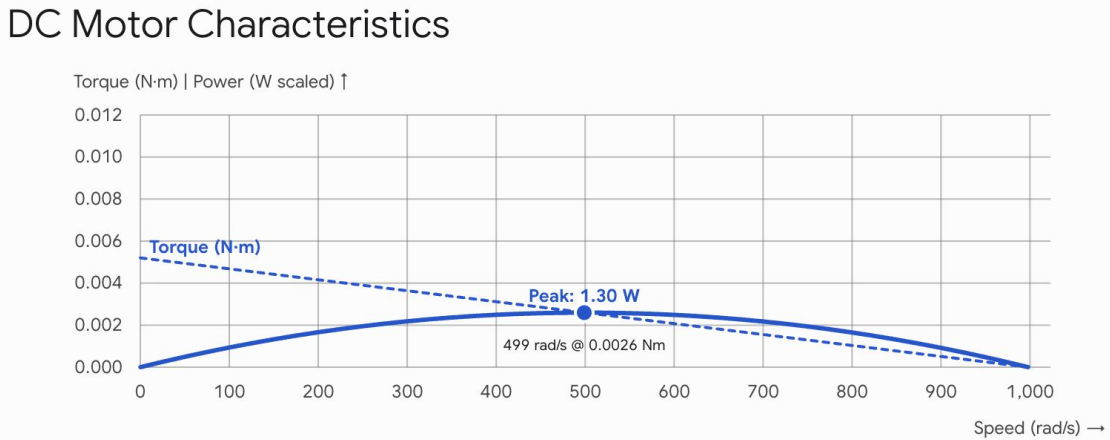
t

Want to optimize for maximum power:

- ½ no load speed = 499 rad/s
- ½ stall torque = 2.6×10^{-3} N·m
- Vappl = 2.5 V

$$T_{load} = T_{em} - T_{fric}$$

$$V_{appl} = V_{em} + i \cdot R$$



Transmission Ratio

Input velocity:
- Set point: **499 rad/sec**

400:1 Gearbox →

Velocity:
- After gearbox: **1.25 rad/sec**

??? →

Output Velocity:
- Match platen speed: **5 mm/sec**



Given: v = w*r

Pinion diameter of 8 mm

We Expect Force to be

1. Net Motor Torque

$T_{load} = T_{em} - T_{fric}$

$T_{load} = 0.00367 - 0.000453\ N{\cdot}m$

$T_{load} = 0.003217\ N{\cdot}m$

2. Output Torque at the Pinion

$t_{out} = T_{load} \cdot Rg \cdot n$

$t_{out} = 0.003217\ N{\cdot}m \times 400 \times 0.334$

$t_{out} = 0.42979\ N{\cdot}m$

3. Upward Force on the Platen

$F_{up} = t_{out} / r_{pinion}$

$F_{up} = 0.42979\ N{\cdot}m / 0.0045\ m$

$F_{up} = 95.5\ N$

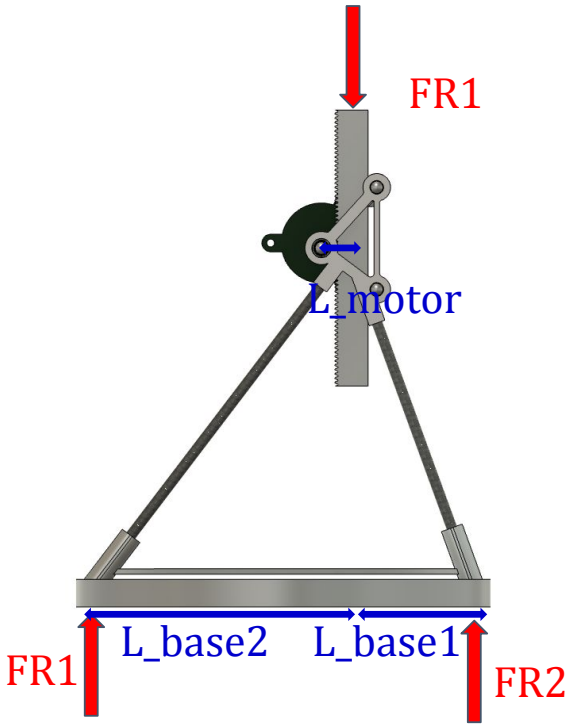
4. Total Downward Force on Platen

$F_{down} = F_{up}$ (equilibrium)

Total Downward Force = 95.5 N

Accounting for friction, the mechanism pushes up on the platen with 95.5 N of force.

Overall FBD



$$\sum F_y = 2F_{R1} + 2F_{R2} - F_M - F = 0$$

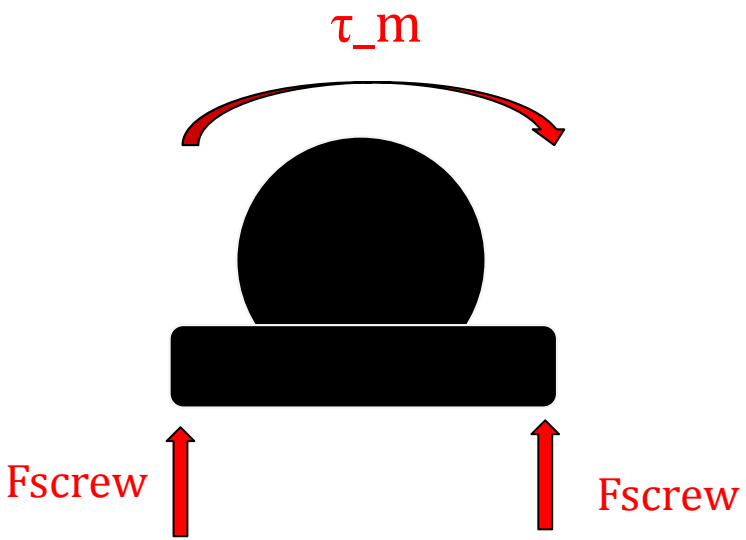
$$\sum M_O = 2F_{R2}(L_{base1}) - 2F_{R1}(L_{base2}) + F_M(L_{motor}) - F(l_R) = 0$$

L_base,1 = L1 sin(26.6°) = 0.054 m

L_base,2 = L2 sin(33.69°) = 0.073 m

F = 95.5 N (design load)

Motor FBD



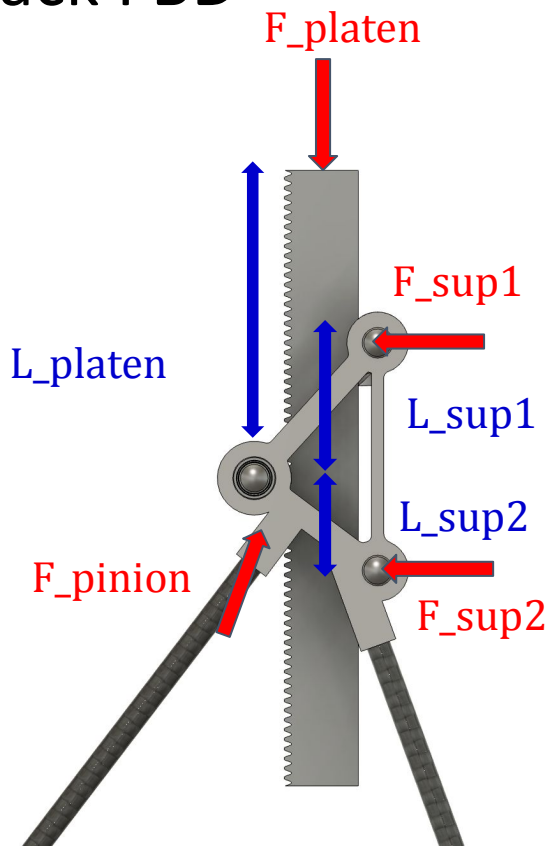
$$\Sigma M_O = -\tau_m + F_{\text{screw},2} = 0$$

$$\Sigma F_x = F_{\text{screw},2} + F_{\text{screw},1} = 0$$

$\tau M = 2.6 \times 10^{-3}$ N·m

lmotor = 0.057 m (motor length)

Rack FBD



$$\Sigma F_x = F_{RP}\sin(\alpha) - F_{sup1} - F_{sup2} = 0$$

$$\Sigma F_y = F_{RP}\cos(\alpha) - F_{platen} = 0$$

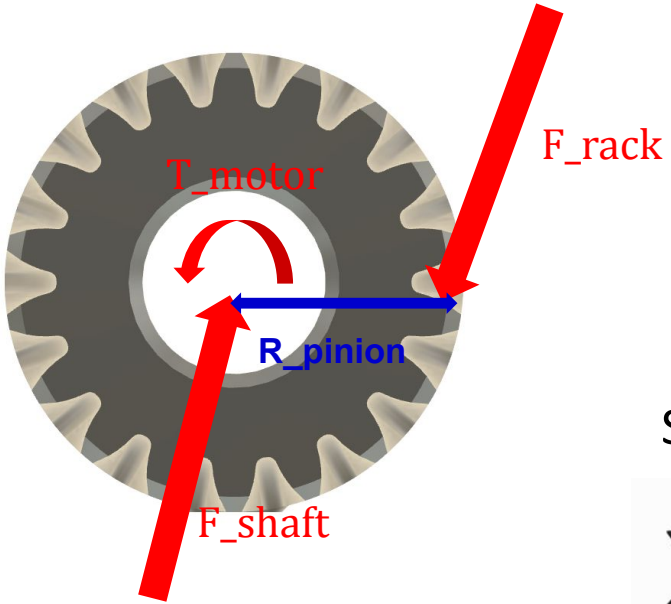
$$\Sigma M_O = -F_{sup1}(l_b) + F\left(\frac{w_r}{2}\right) + F_{RP}\sin(\alpha)(l_{RP}) - F_{RP}\cos(\alpha)(w_R) = 0$$

Rack: 100 mm × 12 mm × 5 mm

wR = 5 mm, L_sup2 and L_sup2 = distance between rollers

α = 20°

Pinion FBD



Sum of Forces

$$\sum F_x = F_{shaft,x} - F_{rack} \sin \alpha = 0$$

$$\sum F_y = F_{shaft,y} - F_{rack} \cos \alpha = 0$$

Sum of Moments

$$\sum M_O = T_{\text{motor}} - F_{\text{rack}} \cos \alpha (R_{\text{pinion}}) = 0$$

Inverse Analysis

$F = 95N \quad \Theta = 33.69° \quad L = 0.1321m \quad I = \frac{\Pi}{4}(r_0^4 - r_i^4) \quad r_i = r_0 - t$

$C = 4 \quad E = 70GPa \quad Sy = 1000MPa \quad t = 0.75mm \quad FoS = 1.5$

bending analysis

$\sigma = \frac{M r_0}{I} \quad \sigma_y = \sigma FoS \quad M = (F/4)sin(\Theta)\, l = 1.958Nm$

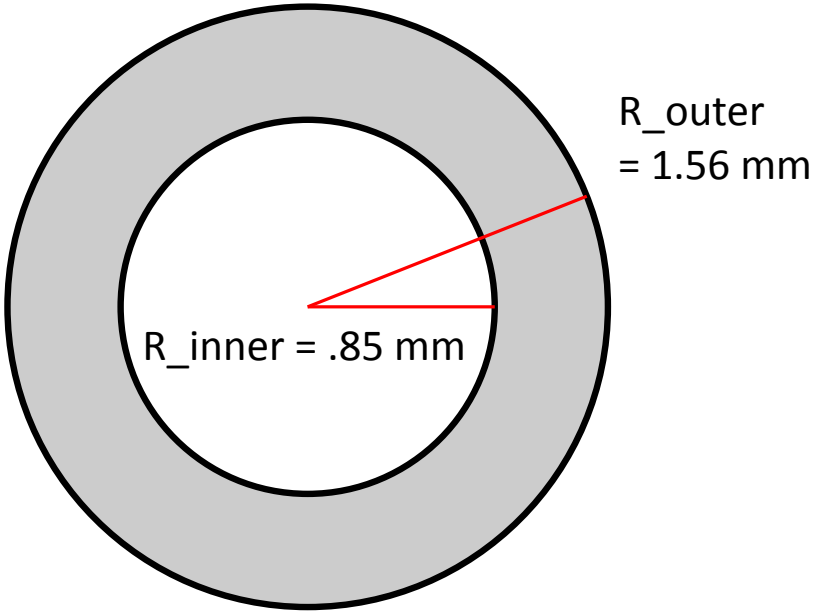
$\sigma = \frac{M r_0}{\frac{\Pi}{4}(r_0^4 - r_i^4)} \Rightarrow r_0 = 1.595mm$

buckling analysis

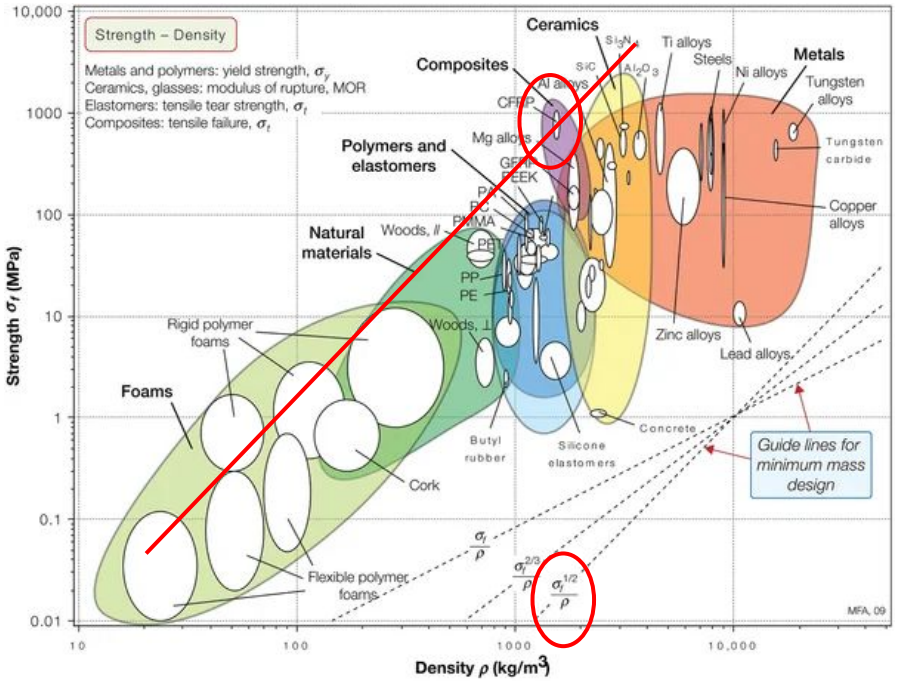
$P = F/4cos\Theta = 32.11N \quad P_{design} = FoS * P = 48.17N$

$I = \frac{C\Pi^2 E}{P_{design} l^2} \Rightarrow r_0 = 0.789mm$

$0.789mm < 1.595mm \Rightarrow D_o = 3.19mm \; D_i = 1.69mm$



Material Selection: Legs



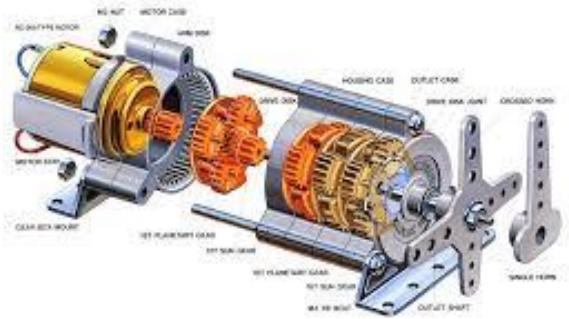
$$m = (4\pi t)^{1/2} \cdot \left(LM^{1/2}\right) \cdot \left(\frac{\rho}{\sigma_f^{1/2}}\right)$$

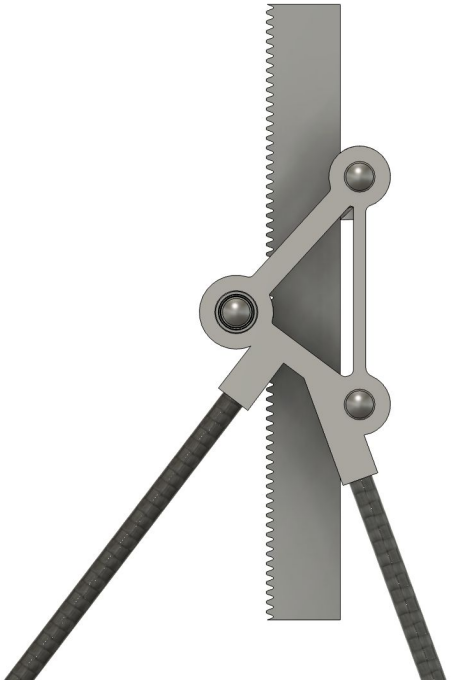
$$m \propto \left(\frac{\rho}{\sigma_f^{1/2}}\right)$$ ← Minimize mass

Transmission Efficiency

Energy Losses:

- Main energy loss due to friction
  - Planetary gearbox
  - Between rack and pinion
  - Between rack and backboard





Transmission Efficiency

Gearbox:

- η = 0.334

Rack and pinion:

- η = 0.96
  - Assuming:
    - μ = 0.2 (steel-steel)
    - α = 20 degrees
    - N = 18 teeth

$$\eta = \frac{1 - \mu(\alpha)}{1 - \mu(\alpha - \frac{\pi}{N_p})}$$

$$\eta_{\text{overall}} = 0.334 \times 0.96 = 0.321 \quad (32.1\%)$$

Transmission Efficiency - Energy Loss

Rolling resistance force: $$F_{rr} = C_{rr} \cdot F_s$$

- $C_{rr} \approx 0.003$ Coefficient of rolling resistance (nylon on steel)
- $F_s = 16.33\ \text{N}$ Normal force on each roller

$$F_{rr,\,\text{total}} = 0.049\ \text{N} \times 2 = \mathbf{0.098}\ \text{N}$$

$$\text{Work Loss} = 0.098\ \text{N} \times 0.1\ \text{m} = \mathbf{0.0098}\ \text{J}$$

Catalog Component Selection



Dry-Running Nylon Sleeve Bearing For 5 Mm Shaft Diameter And 8 Mm Housing Id, 5 Mm Long

6389K702



Dowel Pin, 52100 Alloy Steel, 5 Mm Diameter, 30 Mm Long

91595A372



416 Stainless Steel Dowel Pin
3/4" Diameter, 5" Long

98380A750

Nylon on Steel

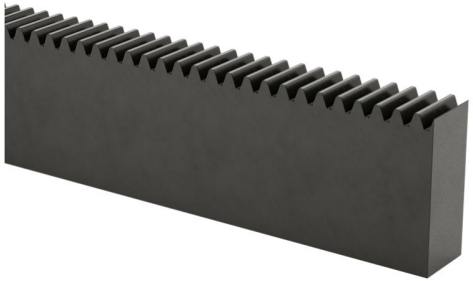
- **Kinetic (μ_k):** 0.15 – 0.30 (Dry)
- **Static (μ_s):** 0.30 – 0.40 (Dry)

Steel on Steel

- **Kinetic (μ_k):** 0.42 – 0.60 (Dry) | 0.05 – 0.16 (Lubricated)
- **Static (μ_s):** 0.74 – 0.80 (Dry) | 0.10 – 0.20 (Lubricated)

Catalog Component Selection





Known: v **= 5 mm/s** velocity

Calculated: ω_{out} **= 1.11 rad/s** (derived from $5/4.5$)

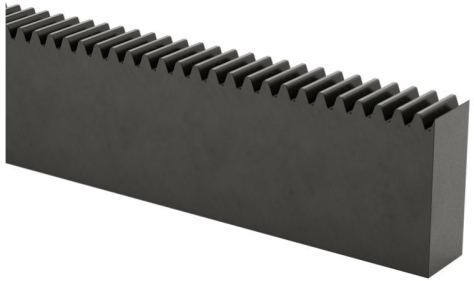
To change rotary to linear motion: d_{pinion} **= 9 mm**

Using FOS = 1.5, T = 100N * 0.0045 m, and N = 20:

- $\mathbf{m_{steel}}$ **= 3.05 g**

Catalog Component Selection





Applied Bending Stress (Metric Lewis Formula)

$$\sigma_{nom} = \frac{F_t}{w \cdot m \cdot Y}$$

$$\sigma_{nom} = \frac{95.51\ \text{N}}{5\ \text{mm} \cdot 0.45\ \text{mm} \cdot 0.32} = 132.65\ \text{MPa}$$

$$132.65\ \text{MPa} \geq 50\ \text{MPa} \implies \textbf{GEAR FAILS}$$

Catalog Component Selection



Column Buckling Analysis

$$FoS_{\text{buckling}} = \frac{F_{\text{cr}}}{F_{\text{applied}}} = \frac{\pi^2 \cdot E_{\text{steel}} \cdot 0.005 \cdot 0.012^3}{48 \cdot 0.05^2 \cdot F_{\text{applied}}}$$

$$= \frac{\pi^2 \times (206 \times 10^9) \times 0.005 \times 0.012^3}{48 \times 0.05^2 \times 100} \gg 1.2$$

Axial Yield Analysis

$$FoS_{\text{axial}} = \frac{S_y}{\sigma} = \frac{S_y \cdot A \cdot E_{\text{steel}}}{F \cdot L}$$

$$= \frac{(530 \times 10^6) \times (0.012 \times 0.005) \times (206 \times 10^9)}{100 \times 0.05} = \frac{530}{4.05 \times 10^{-6}}$$

Operating Current/Heat Calculations

Calculated Thermal Limit: 7.47 A

$$i_{max}^2 = \frac{T_{max} - T_{env}}{R \cdot R_t \cdot (1 - e^{-t/\tau_{th}})}$$

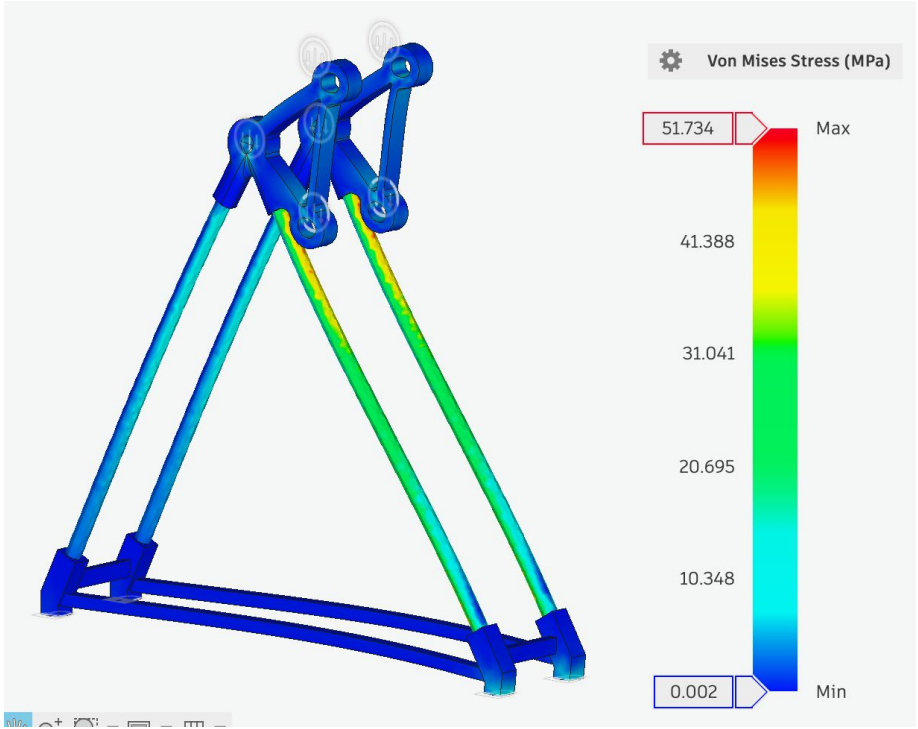
Calculated Operating Current: 0.912 A

$$i_{opt} = \frac{V_{app} - (\omega_{opt} \cdot k)}{R}$$

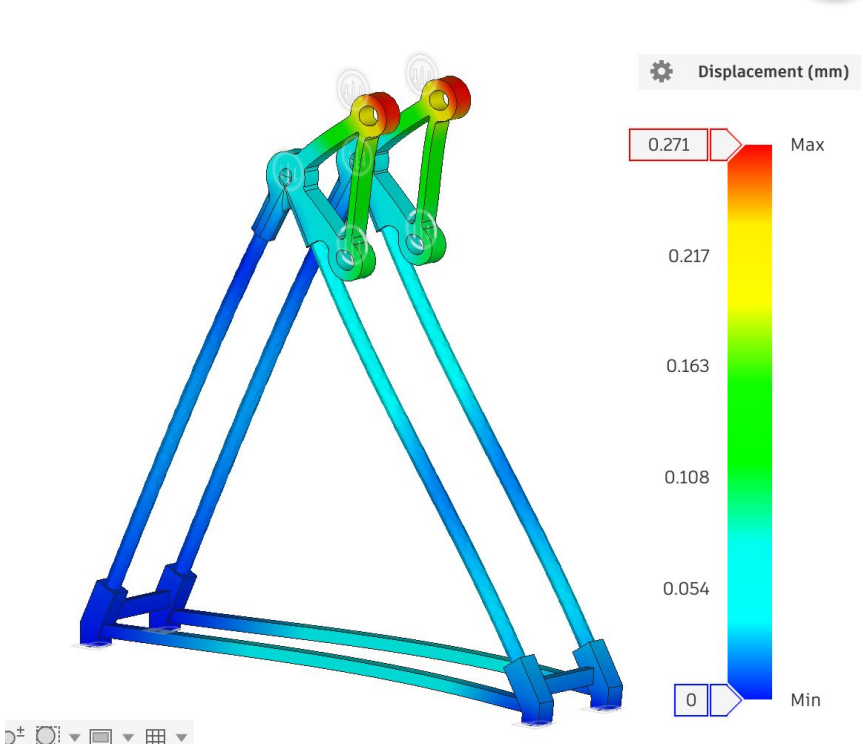
$$i_{opt} \leq i_{max}$$

$$0.912 \text{ A} \leq 7.47 \text{ A}$$

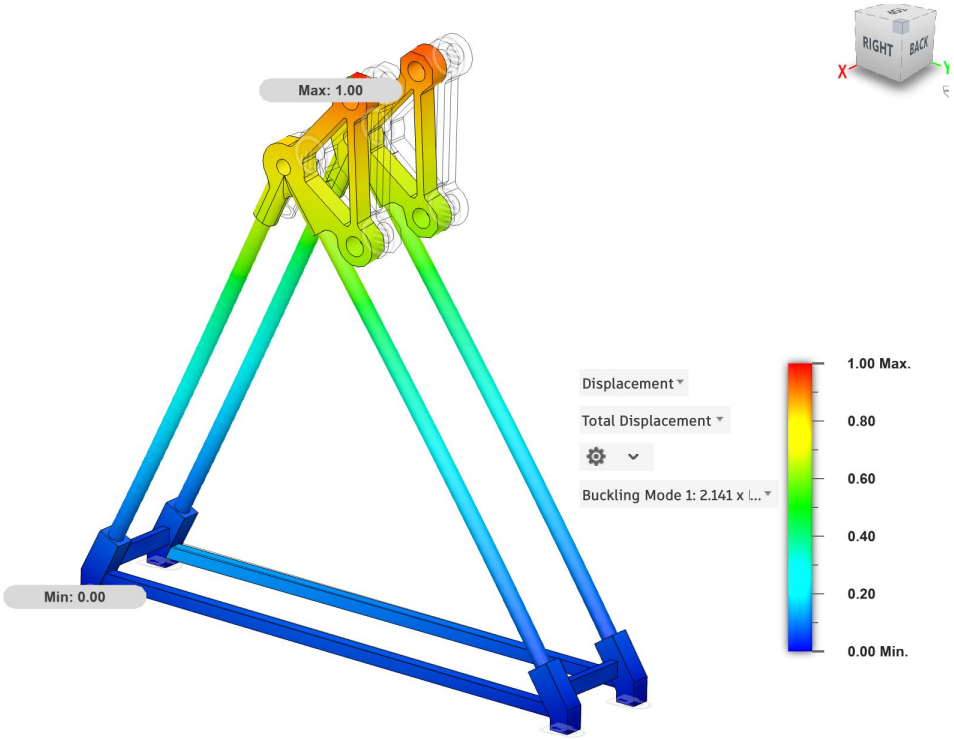
FEA - Static Stress



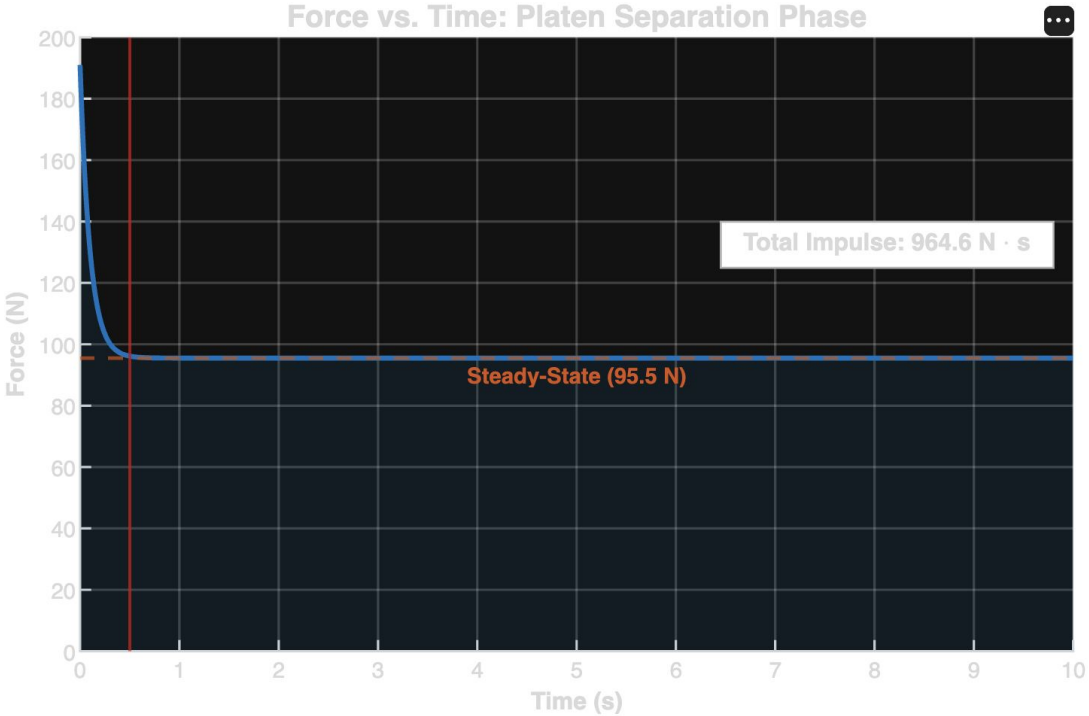
FEA - Displacement



FEA - Buckling



Estimated Outcome



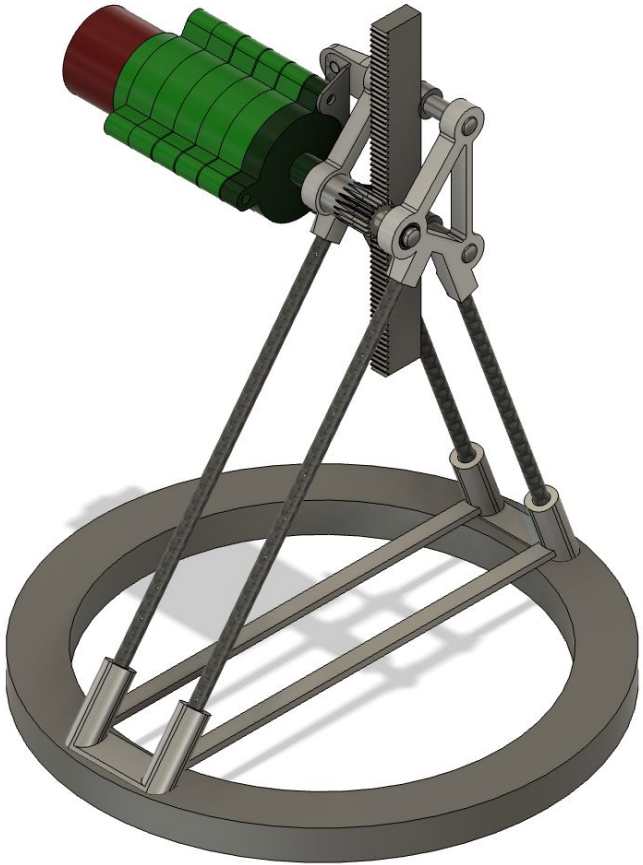
$$\text{Operating Torque} = \frac{1}{2} \cdot T_{stall}$$

$$F_{peak} = F_{steady} \cdot \left(\frac{T_{stall}}{T_{operating}} \right)$$

$$F_{peak} = 95.5\ \text{N} \cdot \left(\frac{3.66 \times 10^{-3}\ \text{N} \cdot \text{m}}{1.83 \times 10^{-3}\ \text{N} \cdot \text{m}} \right)$$

$$F_{peak} = 95.5\ \text{N} \cdot (2)$$

$$F_{peak} = 191\ \text{N}$$



Summary

- **Estimated impulse of 964 N*s**
- **Carbon Fiber legs for mass efficiency**
- **Steel rack and pinion for strength**
- **Nylon bearings to reduce energy loss**