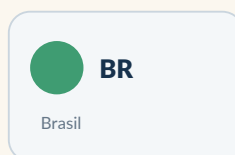
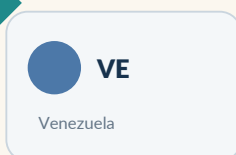
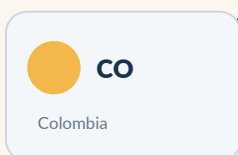
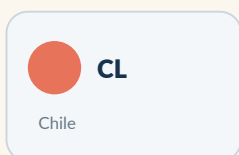
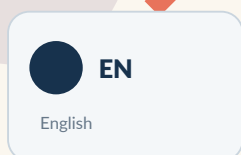




LUMARA ENGLISH

TEACHER EDITION & COMPLETE ANSWER BOOK

Mathematics Through English | CEFR A1-B2



TEACH. SUPPORT. ASSESS.

Complete lesson guidance, language scaffolds, differentiation, listening scripts, worksheet solutions, puzzle keys, quiz answers, assessment keys, rubrics, game notes, projects and feedback tools.

FOR USE WITH THE 112-PAGE STUDENT TEXTBOOK

How to Teach with This Book

Make mathematical thinking audible, visible and discussable.

PRE-TEACH

Select 6-10 essential terms. Use visuals, gestures, examples and multilingual comparison before independent work.

MODEL

Think aloud. Name the operation, explain why it fits, show the calculation, include units and check reasonableness.

REHEARSE

Give learners private thinking time, pair rehearsal and sentence frames before public speaking.

RECAST

Respond to meaning first, then reformulate the learner's sentence accurately without interrupting productive reasoning.

ASSESS

Separate mathematical understanding from English accuracy when appropriate, then provide a next step for each.

Core routine: Notice -> Name -> Model -> Practise -> Explain -> Check -> Reflect. Repeat this language cycle in every unit.

Assessment Rubric

Use the analytic categories separately. A learner may demonstrate strong mathematics with developing English, or fluent English with an incomplete mathematical method.

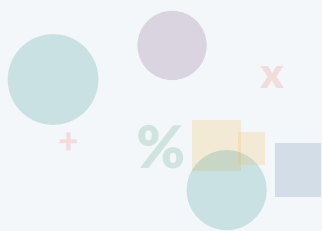
Category	Strong	Developing	Developing	Fluent
Mathematical accuracy	Accurate and complete	Minor slip; method sound	Partial understanding	Major misconceptions
Reasoning	Fully justified and checked	Clear linked reasoning	Some steps unexplained	Little or no reasoning
Vocabulary	Precise and varied	Mostly accurate	Limited or repetitive	Frequent confusion
Communication	Coherent, audience-aware	Generally clear	Meaning sometimes unclear	Difficult to follow
Representation	Selects and evaluates well	Suitable representation	Needs guidance	Representation missing
Units / notation	Consistently accurate	One minor omission	Several omissions	Mostly inaccurate

Feedback formula: Evidence of success + one precise next step + an invitation to revise. Example: "Your graph comparison uses exact values. Now explain how the axis scale affects the visual impression."

Differentiation and Inclusive Practice

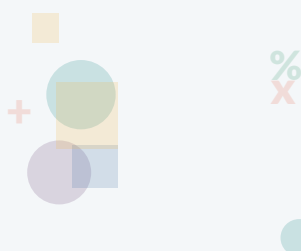
EMERGING ENGLISH

Allow bilingual annotation, word banks, diagrams, matching, oral rehearsal and sentence frames. Reduce language load without reducing mathematical thinking.



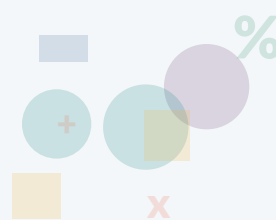
ON-LEVEL

Require complete explanations using target vocabulary, linked steps, units and checking. Use pair roles so every learner speaks.



EXTENSION

Ask learners to compare methods, generalise, identify assumptions, critique representations and design original problems.



ACCESS CHECKLIST

- Readable font and uncluttered page
- Examples before abstraction
- Colour is not the only meaning cue
- Instructions are chunked and modelled
- Thinking time precedes speaking
- Multiple ways to show understanding
- Errors are treated as learning evidence

Never assume that a language error is a mathematics error. Ask the learner to point, draw, model or explain in another way before deciding what they understand.

Suggested Pacing Map

Each unit can be taught in four to eight lessons depending on learner age, English level and mathematical background.

Lesson	Activities	Language focus	Learning objectives
1	Launch + vocabulary	A-C	Recognise and pronounce key terms
2	Concept lab	D	Connect language, visual and method
3	Reading + listening	E-F	Extract mathematical information
4	Pair work + game	G	Explain and negotiate meaning
5	Worksheet + puzzle	H	Apply independently
6	Quiz + challenge	I	Demonstrate, reflect and extend

Homework rhythm: retrieval of five terms -> two calculations -> one explanation -> one self-check. Keep homework short enough that learners can focus on language quality rather than volume.

Unit 1: Teaching Notes

Number Sense and Mathematical Language

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- say and write whole numbers
- understand place value
- compare and order numbers
- use basic symbols in spoken English

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- What number comes next?
- Read the number aloud.
- Which number is greater?
- Round to the nearest ten.
- Write the number in words.

Watch for: Learners may confuse a digit with a number. Spanish and Portuguese decimal commas can transfer into English writing. Reading large numbers requires careful chunking.

Unit 1: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

In all four translation varieties, número and dígito are standard. In English, a digit is one symbol from 0 to 9; a number may contain several digits.

CONCEPT EXPLANATION

Place value changes the meaning of a digit. Compare numbers from the highest place value first.

PLACE VALUE

In 47,286, the digit 7 has a value of 7,000.

Prompt: What changed in this step, and why?

COMPARE

58,340 is greater than 58,304 because 4 tens is greater than 0 tens.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. four thousand five hundred and eighty-two

2. 7,000

3. 94, 409, 490, 904

4. 270

5. 2,500

6. less than

7. 10,000

8. Answers vary.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 1: Reading and Listening Guide

READING ANSWERS

1. 128
2. twenty
3. two hundred
4. 19 calculators
5. 210

LISTENING SCRIPT

Welcome to the LUMARA maths club. We have three hundred and forty-two counters, ninety-five rulers, one thousand and eight flashcards, and sixty-seven calculators. Write each number in digits. Circle the greatest and underline the smallest.

LISTENING ANSWERS

1. 342
2. 95
3. 1,008
4. 67
5. 1,008

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 1: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. 906
2. 500
3. less than
4. 140
5. 65, 506, 560
6. twelve thousand and forty
7. 999
8. 427

PUZZLE KEY

X	Z	R	B	G	M	H	U	N	D	R	E	D	S
R	I	E	I	P	L	I	L	R	S	N	M	A	Q
O	Y	K	R	D	L	C	L	Y	X	Z	B	U	X
J	N	K	K	O	I	A	D	L	K	E	R	H	P
T	J	E	O	D	I	G	C	H	I	C	V	N	N
J	H	V	S	G	P	L	I	E	E	O	U	G	Q
Z	S	O	E	Z	J	C	N	T	V	M	N	O	Q
M	Y	Q	U	T	Y	E	V	R	B	A	E	F	S
R	K	P	Z	S	J	N	C	E	T	H	L	F	W
E	Y	H	N	R	A	K	R	R	Y	O	Y	U	Q
Z	E	J	H	P	I	N	Q	T	E	N	S	G	E
L	A	U	D	K	P	C	D	N	C	Q	M	H	F
C	J	C	J	K	Y	O	K	E	E	X	I	J	M
K	S	H	U	N	D	R	E	D	Q	U	E	N	D

Words may run horizontally, vertically or diagonally.

FACILITATION NOTES

Pair work: Student A reads six numbers. Student B writes them in digits and compares two using greater than, less than or equal to. Change roles.

Game: Human Number Line: learners stand in order from least to greatest without showing their cards. They may only use English comparison phrases.

CHALLENGE RESPONSE

Create a six-digit number in which 4 has a value of 40,000 and 7 has a value of 700. Explain.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 2: Teaching Notes

Addition and Subtraction

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- name parts of addition and subtraction
- read equations aloud
- solve one-step problems
- explain a simple method

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- Add the two numbers.
- Subtract 18 from 50.
- What is the total?
- Find the difference.
- Check using the inverse operation.

Watch for: "Subtract 18 from 50" reverses the spoken number order. Regrouping language may be new even when the method is familiar. Learners may use rest instead of subtract.

Unit 2: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

Resta is widely understood across Chile, Colombia and Venezuela. Sustracción is the more formal school term. Brazilian Portuguese commonly uses subtração.

CONCEPT EXPLANATION

Addition combines quantities. Subtraction removes a quantity or finds a difference. Inverse operations check results.

ADDITION

$368 + 157 = 525$. Add ones, regroup, then add tens and hundreds.

Prompt: What changed in this step, and why?

SUBTRACTION

$702 - 286 = 416$. Regroup one hundred as ten tens, then one ten as ten ones.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. 83

2. 394

3. 43

4. 342

5. 600

6. 38

7. 37

8. Answers vary.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 2: Reading and Listening Guide

READING ANSWERS

1. 38
2. 85
3. They are damaged.
4. 73
5. Add 73 and 12.

LISTENING SCRIPT

The school shop sold twenty-eight notebooks in the morning and thirty-six in the afternoon. It sold nineteen pens, but five were returned. Find the notebook total, the number of pens kept and the combined total.

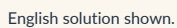
LISTENING ANSWERS

1. 28
2. 36
3. 64
4. 14
5. 78

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

QUIZ ANSWERS

- ## PUZZLE KEY



FACILITATION NOTES

Game: Equation Relay: the next learner begins only after the previous learner says the full equation aloud in English.

CHALLENGE RESPONSE

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 3: Teaching Notes

Multiplication, Division and Order of Operations

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- read multiplication and division equations
- use factors and multiples
- interpret remainders
- apply order of operations

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- Multiply 6 by 8.
- How many equal groups are there?
- Divide 45 by 5.
- What is the remainder?
- Calculate inside the brackets first.

Watch for: Remainders must be interpreted in context. Students may ignore brackets or operation priority. Times is the natural spoken multiplication word.

Unit 3: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

English “times” is spoken in equations: six times seven. “By” also appears in dimensions, such as four by five metres.

CONCEPT EXPLANATION

Multiplication and division describe equal groups and are inverse operations. Apply brackets and operation priority carefully.

MULTIPLICATION

$24 \times 6 = 144$. Think: $(20 \times 6) + (4 \times 6)$.

Prompt: What changed in this step, and why?

DIVISION

$156 / 12 = 13$ because $12 \times 13 = 156$.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. 56

2. 168

3. 12

4. 15 remainder 7

5. 1, 2, 3, 6, 9, 18

6. 7, 14, 21, 28, 35

7. 17

8. 32

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 3: Reading and Listening Guide

READING ANSWERS

1. 8
2. 19
3. 84
4. factors
5. the addition

LISTENING SCRIPT

A sports club has seventy-two bottles shared equally among nine teams. Each receives eight. Four new teams arrive and each needs six. Find the extra bottles and the new total used.

LISTENING ANSWERS

1. 72
2. 9
3. 8
4. 24
5. 96

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 3: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. 54
2. 12
3. 60
4. 9
5. Any valid pair.
6. 8
7. 108
8. 9 remainder 2

PUZZLE KEY

S	A	G	E	X	A	P	M	T	H	X	Q	S	Z
O	M	R	H	X	R	L	U	N	Y	A	U	Z	X
K	Q	R	P	O	U	P	L	W	B	T	O	C	O
U	I	D	D	L	G	M	T	Y	Z	G	T	J	N
U	C	U	I	C	D	W	I	N	R	V	I	Y	Q
M	C	V	Q	V	P	D	P	C	V	X	E	O	W
T	U	H	K	I	I	E	L	B	S	B	N	O	G
M	Z	L	W	Q	Y	S	Y	F	I	S	T	N	H
M	U	L	T	I	P	L	I	C	A	T	I	O	N
J	J	J	L	I	J	J	V	O	N	C	I	A	K
C	C	Z	N	Z	P	V	I	B	N	G	T	P	F
Q	Q	X	Q	X	E	L	N	L	Z	U	U	O	F
U	A	V	R	E	V	X	E	X	A	B	O	L	R
U	T	I	M	E	S	D	I	V	I	D	E	Q	V

Words may run horizontally, vertically or diagonally.

FACILITATION NOTES

Pair work: Array Designer: one partner describes an array; the other draws it and writes multiplication and related division equations.

Game: Factor Hunt: teams list factor pairs for a target number and say them aloud.

CHALLENGE RESPONSE

Use 2, 3 and 4 with different operations or brackets to make three different results.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 4: Teaching Notes

Fractions, Decimals and Percentages

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- name parts of a fraction
- compare equivalent values
- read decimals and percentages
- solve percentage problems

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- The numerator is three.
- The denominator is eight.
- These fractions are equivalent.
- Write the decimal as a percentage.
- Find twenty-five percent of eighty.

Watch for: A larger denominator does not automatically mean a larger fraction. 0.5 and 0.50 are equal. Percent means per hundred.

Unit 4: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

Spanish normally uses a decimal comma in many school contexts; English uses a decimal point. Brazilian Portuguese also commonly uses a decimal comma.

CONCEPT EXPLANATION

Fractions, decimals and percentages are alternative representations of the same proportion or part of a whole.

EQUIVALENT FORMS

$$3/4 = 0.75 = 75\%.$$

Prompt: What changed in this step, and why?

PERCENTAGE

$$25\% \text{ of } 80 = 0.25 \times 80 = 20.$$

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. numerator 5; denominator 8

2. $1/2$

3. 0.75

4. 62%

5. 0.35

6. 30

7. 45%

8. \$30

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 4: Reading and Listening Guide

READING ANSWERS

1. one quarter
2. 0.25
3. 30%
4. vegetables
5. the same part of a whole

LISTENING SCRIPT

In a class of forty, fifty percent choose music, one quarter choose sport and fifteen percent choose art. The rest choose technology. Calculate each group.

LISTENING ANSWERS

1. 40
2. 20
3. 10
4. 6
5. 4

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

QUIZ ANSWERS

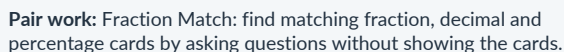
2.7/10

4.0.12

6.30

8.9

FACILITATION NOTES



Game: Percent Shop: teams calculate sale prices from price and discount cards and explain the calculation.

English solution shown.

Design a visual showing that $\frac{2}{5}$, 0.4 and 40% are equivalent.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 5: Teaching Notes

Geometry: Shapes, Angles, Perimeter and Area

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- name 2D and 3D shapes
- describe properties
- classify angles
- calculate perimeter and area

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- How many sides does it have?
- It has four equal sides.
- This angle is acute.
- Calculate the perimeter.
- Find the area in square units.

Watch for: A square is also a rectangle under inclusive definitions. Perimeter uses linear units; area uses square units. A 3D face is different from an edge.

Unit 5: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

“Área” and “perímetro” are direct cognates. Teach pronunciation explicitly so learners do not rely only on spelling.

CONCEPT EXPLANATION

Shape properties help us classify objects. Perimeter measures the boundary; area measures the surface inside.

PERIMETER

Rectangle 8 cm by 5 cm: $P = 2(8 + 5) = 26$ cm.

Prompt: What changed in this step, and why?

AREA

$A = \text{length} \times \text{width} = 8 \times 5 = 40$ cm².

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. pentagon

2. 8

3. acute

4. right

5. 26 cm

6. 36 cm²

7. 4

8. Answers vary.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 5: Reading and Listening Guide

READING ANSWERS

1. rectangle
2. 56 metres
3. 180 square metres
4. circle, triangle or square
5. shapes, angles and symmetry

LISTENING SCRIPT

Draw a rectangle eight centimetres long and five wide. Inside it, draw a square with two-centimetre sides. Above it draw an acute angle. To the right draw a circle with radius three centimetres. Label everything.

LISTENING ANSWERS

1. 8 cm by 5 cm
2. 2 cm
3. acute
4. 3 cm
5. all shapes and measurements

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 5: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. 6
2. cone
3. obtuse
4. 28 cm
5. 49 cm²
6. 6
7. a point where sides or edges meet
8. circle or sphere

PUZZLE KEY

V	X	P	O	L	Y	G	O	N	Y	C	Y	S	Q
E	Q	Q	R	Q	X	W	T	I	H	S	X	V	N
X	O	V	K	T	T	R	I	A	N	G	L	E	U
U	H	E	H	W	G	E	O	M	E	T	R	Y	N
T	V	R	P	J	H	J	P	V	L	Q	P	R	E
P	V	T	I	O	G	F	G	P	K	G	S	E	K
C	H	E	T	G	S	K	D	B	S	Q	T	C	A
V	E	X	Z	S	N	C	B	H	U	A	F	T	M
I	R	P	C	X	A	Z	A	A	A	F	S	A	C
I	E	B	A	G	M	P	R	F	E	K	E	N	E
C	I	R	C	L	E	E	J	D	W	W	J	G	D
C	J	Y	H	C	C	F	A	C	E	X	K	L	G
F	D	Q	B	E	O	I	D	Y	Y	Y	W	E	E
H	D	Z	H	Y	I	Z	U	X	P	U	F	C	C

Words may run horizontally, vertically or diagonally.

FACILITATION NOTES

Pair work: Mystery Shape: ask yes/no questions about sides, angles, faces, edges or vertices and guess the shape.

Game: Geometry Gallery Walk: match shape cards to property cards and justify each match in English.

CHALLENGE RESPONSE

Design two rectangles with area 24 square units. Compare their perimeters.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 6: Teaching Notes

Measurement: Length, Mass, Capacity, Time and Temperature

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- select appropriate metric units
- read measuring instruments
- convert common units
- describe measurements precisely

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- How long is it?
- Which unit is most appropriate?
- The scale increases by...
- Convert metres to centimetres.
- The temperature rose by five degrees.

Watch for: Mass and weight are often used interchangeably in everyday speech, but they are different scientific quantities. Learners may multiply when they should divide during conversions. English time expressions can differ from digital notation.

Unit 6: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

The textbook uses British spellings metre and litre. American materials may use meter and liter. The mathematical values are identical.

CONCEPT EXPLANATION

Choose a unit that matches the size of the object. Use multiplication to convert from a larger metric unit to a smaller one and division to convert back.

CONVERSION

2.4 m = 240 cm because $2.4 \times 100 = 240$.

Prompt: What changed in this step, and why?

ELAPSED TIME

09:35 to 11:10 = 1 hour 35 minutes.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. 300 cm

2. 2,500 g

3. 1.5 L

4. centimetres

5. 1 hour 45 minutes

6. 7 degrees Celsius

7. 8 pieces

8. Answers vary.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 6: Reading and Listening Guide

READING ANSWERS

1. 12 m
2. 8 m
3. 25 kg
4. 1 hour 30 minutes
5. 5 degrees Celsius

LISTENING SCRIPT

Measure carefully. The blue ribbon is one metre and twenty centimetres long. The red ribbon is eighty-five centimetres long. The bottle contains seven hundred and fifty millilitres. The parcel has a mass of two kilograms and three hundred grams. The classroom temperature is twenty-one degrees Celsius.

LISTENING ANSWERS

1. 1 m 20 cm
2. 85 cm
3. 750 mL
4. 2 kg 300 g
5. 21 degrees Celsius

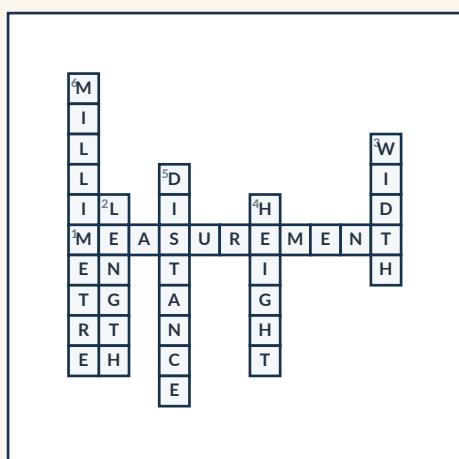
Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 6: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. kilometres
2. 240 cm
3. 3,000 g
4. 2.25 L
5. 1 hour 25 minutes
6. 24 degrees Celsius
7. thermometer
8. Kilograms are a more appropriate larger unit.

PUZZLE KEY



English solution shown.

FACILITATION NOTES

Pair work: Measurement Detective: Student A describes an object and gives a realistic estimate. Student B chooses the best unit, asks one follow-up question and records the estimate. Then measure and compare.

Game: Unit Corners: label four classroom corners mm, cm, m and km. Read an object or distance. Teams move to the most suitable unit and justify their choice in English.

CHALLENGE RESPONSE

Plan a one-day class trip. Include distances, departure and arrival times, water capacity and estimated mass of supplies. Show at least three conversions.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 7: Teaching Notes

Algebra: Patterns, Expressions, Equations and Inequalities

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- identify variables and constants
- evaluate expressions
- solve one-step and two-step equations
- explain algebraic reasoning

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- Let x represent the unknown.
- Substitute the value into the expression.
- Do the same operation on both sides.
- The sequence increases by...
- The solution satisfies the equation.

Watch for: The equals sign means both sides have the same value; it does not mean "the answer comes next." Combining unlike terms is not valid. A negative sign and a subtraction operation need careful reading.

Unit 7: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

Spanish ecuación and Portuguese equação are useful cognates. English expression and equation are not synonyms.

CONCEPT EXPLANATION

An equation is like a balanced scale: whatever operation you perform on one side must also be performed on the other. A variable represents a number that can change or is unknown.

SOLVE

$3x + 5 = 20$; subtract 5: $3x = 15$; divide by 3: $x = 5$.

Prompt: What changed in this step, and why?

CHECK

Substitute 5: $3(5) + 5 = 20$. The solution is correct.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. 23

2. $5a$

3. 17

4. 9

5. 28

6. 7

7. 27, 32

8. $2n + 5$

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 7: Reading and Listening Guide

READING ANSWERS

1. 8
2. 3 tiles
3. the design number
4. 14
5. to verify that the rule works

LISTENING SCRIPT

Solve each equation and write the value of the variable. Number one: x plus seven equals nineteen. Number two: three y equals twenty-four. Number three: z divided by five equals six. Number four: two a plus four equals eighteen. Number five: b minus nine is greater than twelve.

LISTENING ANSWERS

1. 12
2. 8
3. 30
4. 7
5. b is greater than 21

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 7: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. 6
2. 17
3. 23
4. 9
5. 6
6. x is less than 10
7. 24
8. Replace the variable with the proposed value and verify that both sides are equal.

PUZZLE KEY

S	U	B	S	T	I	T	U	T	E	S	K	A	I
L	J	V	U	X	J	L	O	Q	D	K	U	C	N
Q	S	K	I	Z	Z	D	Q	W	Y	C	D	O	E
E	Q	U	A	T	I	O	N	C	O	A	I	N	Q
B	D	E	J	K	V	U	P	E	L	C	R	S	U
U	O	A	Y	B	Q	V	F	G	Q	F	Q	T	A
P	E	G	J	H	X	F	E	W	F	W	S	A	L
T	E	R	M	A	I	B	U	L	S	V	C	N	I
V	T	Y	F	C	R	A	E	U	T	U	H	T	T
A	S	Z	I	A	A	D	D	P	K	F	F	C	Y
Z	F	E	X	P	R	E	S	S	I	O	N	A	Z
G	N	F	Z	P	A	V	A	R	I	A	B	L	E
T	Q	U	N	K	N	O	W	N	G	M	J	U	I
T	H	I	V	X	P	X	B	R	Z	S	D	Z	R

Words may run horizontally, vertically or diagonally.

FACILITATION NOTES

Pair work: Equation Coach: Student A solves an equation aloud but deliberately omits one reasoning step. Student B identifies the missing step and checks the solution by substitution. Change roles.

Game: Algebra Relay: teams receive expression, equation, solution and verbal-description cards. They race to create correct sets and then read each set aloud.

CHALLENGE RESPONSE

Create a visual pattern with a linear rule. Draw Designs 1 to 4, write an expression for Design n and prove that your rule works.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 8: Teaching Notes

Statistics and Probability

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- describe data sets
- calculate measures of centre and spread
- interpret simple probability
- communicate conclusions cautiously

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- The mean is calculated by...
- The median is the middle value.
- The most frequent value is...
- The probability is three out of ten.
- The sample may not represent the population.

Watch for: Mean, median and mode are not interchangeable. Probability should be between 0 and 1, or between 0% and 100%. A larger sample is not automatically unbiased.

Unit 8: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

Promedio is common in everyday Spanish; media aritmética is more formal. Portuguese uses média.

CONCEPT EXPLANATION

Statistics summarises data, while probability describes how likely an outcome is. Always consider how data was collected before making a conclusion.

MEAN

For 4, 7, 7, 10: mean = $28 / 4 = 7$.

Prompt: What changed in this step, and why?

PROBABILITY

3 blue out of 8 equal outcomes: $P(\text{blue}) = 3/8$.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

- 7
- 9
- 4
- 13
- $3/8$
- 0 or impossible
- impossible, unlikely, likely, certain
- Answers vary; for example, the sample was not selected randomly.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 8: Reading and Listening Guide

READING ANSWERS

1. 18
2. cycling
3. 25 minutes
4. 35 minutes
5. The sample is small and may not represent the whole city.

LISTENING SCRIPT

A spinner has eight equal sections: three blue, two green, two yellow and one red. The probability of blue is three eighths. Green and yellow are equally likely. Red is the least likely colour. It is impossible to land on purple because there is no purple section.

LISTENING ANSWERS

1. 8
2. $\frac{3}{8}$
3. green and yellow
4. red
5. There is no purple section.

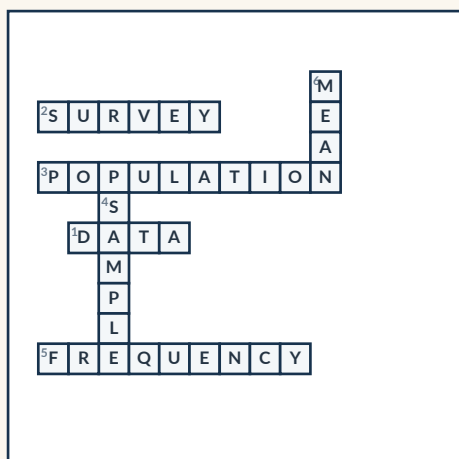
Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 8: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. 7
2. 7
3. 3
4. 17
5. $1/2$
6. $1/5$
7. the complete group being studied
8. An extreme value changes the total and therefore changes the mean.

PUZZLE KEY



English solution shown.

FACILITATION NOTES

Pair work: Mini Survey: choose one safe classroom question, collect ten responses, make a frequency table and tell your partner one conclusion and one limitation.

Game: Probability Line: place event cards from impossible to certain. Teams must defend any disputed placement using likely, unlikely, equally likely and certain.

CHALLENGE RESPONSE

Design a fair game using a spinner or dice. State all outcomes, calculate each probability, explain why the game is fair and test it at least 30 times.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 9: Teaching Notes

Graphs, Tables and Data Interpretation

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- identify parts of graphs
- select suitable displays
- compare trends and categories
- evaluate misleading representations

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- The graph shows that...
- There was a steady increase.
- The value reached a peak of...
- By contrast, the second category...
- The scale may exaggerate the difference.

Watch for: A steep-looking line does not always mean a large numerical change; the scale matters. Correlation in two trends does not by itself prove causation. Pie-chart sectors should represent a complete whole.

Unit 9: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

English uses graph and chart flexibly. In Spanish and Portuguese, gráfico is a broad equivalent; tabla/tabela refers specifically to a table.

CONCEPT EXPLANATION

A graph should communicate data clearly and honestly. Read the title, labels, units, scale and source before interpreting patterns or comparing values.

ABSOLUTE CHANGE

From 80 to 100: increase = 20.

Prompt: What changed in this step, and why?

PERCENTAGE CHANGE

$20 / 80 \times 100 = 25\%$ increase.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. horizontal and vertical axes
2. line graph
3. bar chart
4. 20
5. 25%
6. the highest point or value
7. It can exaggerate visual differences.
8. Answers vary.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 9: Reading and Listening Guide

READING ANSWERS

1. 420 units
2. June
3. 390 units
4. The vertical axis begins at 350 instead of zero.
5. There was a real increase, but the display exaggerates it.

LISTENING SCRIPT

Study the sales graph. Sales begin at forty units in January, rise gradually to sixty-five in March, remain stable in April, fall sharply to thirty in May and recover to fifty-five in June. The greatest increase occurs between February and March. The lowest value occurs in May.

LISTENING ANSWERS

1. 40 units
2. Sales remain stable.
3. 30 units
4. May
5. Sales recover or increase by 25 units.

Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 9: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. what the data represents
2. the value represented by each interval
3. 30
4. 20%
5. pie chart
6. a consistent change in one direction
7. Answers may include axis origin, unequal intervals, missing labels, distorted images or selective data.
8. Answers vary.

PUZZLE KEY

V	T	A	N	I	S	D	B	W	K	G	V	Z	R
T	T	S	U	I	C	C	R	Z	P	N	E	L	W
L	Q	H	C	T	Q	L	F	I	W	S	R	A	D
Z	E	T	J	A	A	C	E	L	T	Q	T	B	C
K	B	J	Z	B	L	C	H	H	U	O	I	A	O
L	S	Q	E	O	H	E	B	A	E	T	C	R	I
Y	I	L	Y	A	L	F	X	V	R	D	A	C	N
B	H	N	R	S	P	Y	C	L	W	T	L	H	T
T	Y	T	E	F	T	W	C	C	M	N	A	A	E
P	Q	W	K	G	N	N	B	N	T	Y	X	R	R
P	T	X	M	J	R	C	N	A	M	T	I	T	V
E	G	U	V	K	T	A	B	W	W	G	S	O	A
Y	Y	T	X	Y	E	L	P	W	D	G	W	A	L
S	H	K	Z	F	E	F	I	H	T	H	A	D	H

Words may run horizontally, vertically or diagonally.

FACILITATION NOTES

Pair work: Information Gap Graph: Student A receives values for January to June; Student B has the blank graph. Ask and answer precise questions, then compare the completed graph with the original.

Game: Graph Gallery Walk: teams match data stories, graphs and interpretations. One card in each set is misleading; identify it and explain why.

CHALLENGE RESPONSE

Create two graphs from the same data: one fair and one intentionally misleading. Explain exactly which design choice changes the reader's impression.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Unit 10: Teaching Notes

Word Problems and Academic Mathematical Communication

LESSON PURPOSE

Learners develop mathematical understanding and the English needed to name, interpret, explain, question and verify that understanding.

OBJECTIVES

- decode multi-step problems
- justify methods with evidence
- present mathematical arguments
- evaluate and revise solutions

PRIOR KNOWLEDGE

Ask learners to solve one familiar example nonverbally, then invite them to label what they did in any language. Build the English terms onto existing understanding.

LANGUAGE FOCUS

- The relevant information is...
- I represented the situation by...
- This method is efficient because...
- The result is reasonable since...
- A limitation of this model is...

Watch for: Learners may calculate every number in a problem rather than select relevant information. Rounding too early can reduce accuracy. A mathematically correct result can still be unreasonable in context.

Unit 10: Vocabulary and Concept Support

REGIONAL LANGUAGE NOTE

Encourage learners to translate the situation, not every word. They should identify quantities, relationships, units and the question.

CONCEPT EXPLANATION

Strong problem solving combines comprehension, representation, calculation, checking and communication. A correct number without a clear interpretation may not fully answer the question.

MODEL

Taxi cost: $C = 3 + 1.20k$, where k is kilometres.

Prompt: What changed in this step, and why?

INTERPRET

For 8 km, $C = 12.60$. The journey costs \$12.60.

Prompt: What changed in this step, and why?

WORKSHEET ANSWERS

1. 7 cards

2. \$60

3. 57 m²

4. \$12.60

5. 750 g

6. 150 L

7. For example, use a proportion or check that $750/10$ equals $300/4$.

8. The tank contains 150 litres of water.

Accept equivalent forms and mathematically valid alternative methods. Require units when the question measures a real quantity.

Unit 10: Reading and Listening Guide

READING ANSWERS

1. 180
2. \$860
3. \$270
4. \$-40, a loss of \$40
5. Attendance is uncertain, so the team needs to understand the risk.

LISTENING SCRIPT

A school orders twelve boxes of notebooks. Each box contains twenty-four notebooks and costs thirty-six dollars. Delivery costs eighteen dollars. The school receives a ten percent discount on the cost of the boxes, but not on delivery. Calculate the number of notebooks, the discount, and the final amount paid.

LISTENING ANSWERS

1. 288
2. \$432
3. \$43.20
4. \$388.80
5. \$406.80

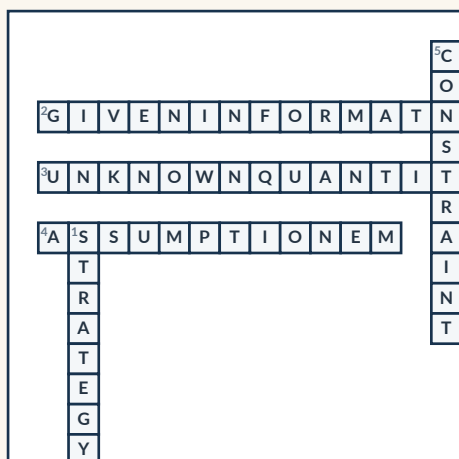
Delivery: Read once at natural pace, pause for pair comparison, read a second time with natural chunking, then display or read the script for self-correction.

Unit 10: Quiz, Puzzle and Activity Key

QUIZ ANSWERS

1. \$45
2. 156 km
3. 12
4. 17
5. 9/10
6. 140 tiles
7. Answers vary; for example, fuel price remains constant.
8. Units identify what the number measures and make the interpretation complete.

PUZZLE KEY



English solution shown.

FACILITATION NOTES

Pair work: Problem-Solving Conference: Student A presents a multi-step solution. Student B asks about the representation, assumptions, units and checking method, then gives one specific improvement.

Game: Maths Escape Challenge: teams solve five linked problems. Each answer unlocks a code. Teams earn an extra clue by explaining one solution clearly in academic English.

CHALLENGE RESPONSE

Investigate a real decision such as class-event costs, travel time or resource use. Build a model, state assumptions, compare two scenarios and present a recommendation supported by mathematics.

Look for a valid model, accurate mathematics, clear language, checking and a justified conclusion. Responses will vary.

Midterm Test Answer Key

PART A | QUESTIONS 1-12

- | | |
|---|-----------|
| 1. seven thousand four hundred and five | 2. 8,000 |
| 3. 6,700 | 4. 6,245 |
| 5. 4,326 | 6. 322 |
| 7. 72 | 8. 29, 35 |
| 9. $\frac{5}{8}$ | 10. 62% |
| 11. 0.35 | 12. 90 |

PART B | QUESTIONS 13-24

- | | |
|--|--|
| 13. pentagon | 14. acute |
| 15. right | 16. 38 cm |
| 17. 84 cm ² | 18. 9 cm |
| 19. cube | 20. $\frac{5}{8}$ |
| 21. \$102 | 22. Perimeter is distance around; area is surface covered. |
| 23. Examples: 4 x 6 gives perimeters 20 and 2 x 12 gives 28. | 24. $\frac{5}{8} = 0.625$, so it is greater than 0.6. |

Extended response: Area = $8 \times 6 = 48$ m². Cost = $48 \times \$18 = \864 . Award full credit for a clear method, correct units and a complete interpretation.

Final Assessment Key: Parts A-C

PART A

1. Definitions should be accurate and contextual.
2. nine hundred and five thousand and seventy
3. 49,000
4. 4,356
5. 5,850
6. 132
7. 0.35 and 35%
8. 42
9. 0.405, 0.45, 48%, $\frac{1}{2}$
10. 15%

PART B

11. 31
12. $7a - 2$
13. 9
14. 42
15. 18, 22; rule $4n - 2$
16. $P = 27 \text{ m}$; $A = 38 \text{ m}^2$
17. 3,750 m
18. 2.45 L
19. 15:40
20. 17 degrees Celsius
21. 52 degrees
22. Area measures two dimensions, so square units are required.

PART C

23. Thursday
24. 18 books
25. Overall increase with small fluctuations; exact wording varies.
26. 11
27. median 10; mode 10; range 7
28. $\frac{1}{4}$
29. A high starting point magnifies the visual height difference
30. One class may not represent the wider population

For open comparison and conclusion items, award credit for accurate evidence, clear language and a defensible interpretation.

Final Assessment Key: Parts D-F

PART D | MULTI-STEP PROBLEMS

31. School event

Revenue: $96 \times 7.50 + 144 \times 4.25 = \$1,332$. Costs: $\$500 + \$185 + (240 \times \$1.20) = \973 . Profit = $\$359$.

32. Water use

Initial water = $0.35 \times 600 = 210$ L. Final water = $210 + 180 - 75 = 315$ L. Percentage full = $315/600 \times 100 = 52.5\%$.

33. Travel model

First 126 km at 70 km/h = 1.8 h. Remaining 189 km at 90 km/h = 2.1 h. Total = 3.9 h, or 3 h 54 min.

34. Evaluate

Answers vary. Credit a relevant assumption or limitation, a plausible effect and a useful improvement.

PART E | SPEAKING

Use the speaking rubric on page 49. Do not penalise accent. Assess intelligibility, vocabulary, sequencing, mathematical accuracy, evidence and response to questions.

PART F | WRITING

Reduction = $120/800 \times 100 = 15\%$. Target use = 75% of 800 = 600 units. Target not met; a further reduction of 80 units is required. Recommendations and limitation vary.

Speaking Assessment Rubric

Criteria	4	3	2	1
Mathematical accuracy	Complete and accurate	Mostly accurate	Partly accurate	Major errors
Vocabulary	Precise and varied	Appropriate	Limited	Frequently inaccurate
Organisation	Clear, logical, cohesive	Mostly clear	Some jumps	Hard to follow
Explanation	Justifies and checks	Explains main steps	Lists steps	Little explanation
Interaction	Responds flexibly	Answers questions	Needs prompting	Minimal response
Intelligibility	Consistently clear	Generally clear	Sometimes unclear	Often unclear

Suggested weighting: Mathematical accuracy 30% | Explanation 25% | Vocabulary 15% | Organisation 15% | Interaction 10% | Intelligibility 5%.

Writing Assessment Rubric

Criteria	4 (Exceeds Expectations)	3 (Meets Expectations)	2 (Approaching Expectations)	1 (Below Expectations)	0 (No Evidence)
Content	Fully answers all parts	Answers most parts	Some parts missing	Task largely unmet	
Mathematics	Accurate, complete, checked	Sound with minor slip	Partial method	Major errors	
Reasoning	Justifies choices and limitations	Links main steps	Some explanation	Minimal reasoning	
Language	Precise vocabulary and cohesive sentences	Generally clear and accurate	Limited range; errors distract	Meaning often unclear	
Notation / units	Consistently correct	One or two slips	Several omissions	Mostly absent	

FEEDBACK STEMS

- Your calculation is clear because...
- Your evidence supports the conclusion when...
- Revise the sentence so that the unit is explicit.
- Add a check to show that the result is reasonable.
- Explain how the limitation affects confidence in the conclusion.

VOCABULARY RELAY

Teams match English terms, translations, symbols, diagrams and examples.

BACK-TO-BOARD

One learner faces away from a displayed term while teammates define it without saying it.

ERROR DETECTIVE

Display a plausible wrong solution. Teams identify, explain and correct the first error.

FOUR CORNERS

Assign four possible answers or units to corners. Learners move, discuss and defend.

MATHS TABOO

Explain a target term without using three obvious related words.

INFORMATION GAP

Partners hold different data and must ask precise questions to complete a table or graph.

HUMAN GRAPH

Learners position themselves as data points or categories, then describe the display.

REASONING AUCTION

Teams receive points and bid on solutions they believe are correct, then justify bids.

Keep competition low-stakes and explanation-rich. Award points for accurate language, useful questions and checking - not only speed.

Listening Delivery and Adaptation

1

FIRST READ - Natural pace. Learners listen only or record key quantities.

2

PAIR CHECK - Learners compare what they heard and identify missing information.

3

SECOND READ - Pause at meaningful chunks, not after every word.

4

SCRIPT CHECK - Display or reread the script. Learners self-correct in another colour.

5

LANGUAGE NOTICE - Underline number forms, units, comparison language and sequencing phrases.

6

TRANSFER - Change one or two values and ask learners to create a new question.

Do not turn listening into memory testing. Permit note-taking, predict the answer type and replay or reread when the learning goal is mathematical language rather than one-time comprehension.

Regional Translation and Notation Notes

The tables prioritise widely understood school vocabulary. Individual schools and teachers may prefer alternatives. Treat variation as a resource for comparison, not as an error.

CHILE

Promedio is very common for mean. Resta is common for subtraction. Decimal comma is frequently used.

COLOMBIA

Promedio and media may both appear. Procedimiento is a common classroom word for working or method.

VENEZUELA

Everyday and formal terminology can vary by school. Encourage learners to share familiar alternatives.

BRAZIL

Portuguese uses equação, média, fração and porcentagem. Decimal comma is common; English target work uses a point.

ENGLISH

This edition uses British metre/litre spellings. “Maths” is British; “math” is American. Both refer to mathematics.

Invite learners to add local alternatives in the margins, then agree on the English term required for the current lesson.

Projects, Homework and Extension

A1 NUMBER MUSEUM

Photograph or draw numbers in daily life. Label place value, units and purpose.

Evidence: maths + English explanation + visual + reflection

A2 MEASUREMENT AUDIT

Measure a room or garden, convert units and present a scaled plan.

Evidence: maths + English explanation + visual + reflection

B1 CLASS SURVEY

Design a fair question, collect data, calculate statistics and make two graphs.

Evidence: maths + English explanation + visual + reflection

B1 ALGEBRA PATTERN GALLERY

Create visual sequences, derive rules and challenge classmates to predict terms.

Evidence: maths + English explanation + visual + reflection

B2 DECISION MODEL

Compare two real options using costs, percentages, assumptions and a recommendation.

Evidence: maths + English explanation + visual + reflection

B2 MISLEADING GRAPH INVESTIGATION

Find or create a misleading display, explain its effect and redesign it fairly.

Evidence: maths + English explanation + visual + reflection

Homework should strengthen retrieval and explanation. Avoid assigning long repetitive sets when a shorter task can reveal understanding more clearly.

Feedback and Reporting Templates

QUICK VERBAL FEEDBACK

You used ___ accurately. Your method was clear when ___. Now improve ___ by ___.

WRITTEN FEEDBACK

Mathematical strength: _____ | English strength: _____ | Next mathematical step: _____ | Next language step: _____

FAMILY / LEARNER REPORT

Current CEFR communication band: ___ | Secure topics: ___ | Developing topics: ___ | Recommended practice: ___

SELF-REFLECTION

I can now... | I still need help with... | A useful phrase is... | My next goal is...

Feedback should be specific enough that the learner knows what to do next. "Be clearer" is not a next step; "name the operation and include the unit" is.

Teacher Reflection and Completion

Your teaching notes

REFLECTION 1

Which visuals or multilingual comparisons unlocked understanding?

REFLECTION 2

Which mathematical terms need more retrieval practice?

REFLECTION 3

Who can reason mathematically but needs English scaffolding?

REFLECTION 4

Which misconceptions appeared, and how will you address them?

REFLECTION 5

What will you adapt before teaching this unit or course again?

Thank you for helping learners discover that mathematical ideas can travel across languages - and that their multilingual knowledge is an academic strength.